

# An EM-Based Estimation of OFDM Signals<sup>1</sup>

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**Abstract** — We propose an EM-based algorithm to efficiently detect transmitted data in an OFDM system as well as estimating the channel impulse response (CIR). The maximum likelihood estimate of CIR is obtained by using channel statistics (their means and covariances) via the expectation-maximization (EM) algorithm. This algorithm can improve signal detection and the channel estimation accuracy by making use of pilot symbols to obtain an initial estimate for the iteration. Simulation results show that the bit error rate (BER) can be significantly reduced by this algorithm, and validate its good convergence and robust properties.

## I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1], a spectrally efficient form of FDM, divides its allocated channel spectrum into several parallel sub-channels. OFDM is inherently robust against frequency selective fading, since each sub-channel occupies a relatively narrow band, where the channel frequency characteristic is nearly flat. OFDM has an additional advantage of being computationally efficient because the fast Fourier transform (FFT) technique can be used to implement the modulation and demodulation functions [2]. Furthermore, the OFDM system can eliminate intersymbol interference (ISI) through use of a cyclic prefix (CP) that must be longer than the length of the channel impulse response (CIR). Figure 1 shows a schematic diagram of an OFDM system. OFDM has already been used in European digital audio broadcasting (DAB), digital video broadcasting (DVB) systems and high performance radio local area network (HIPER-LAN). It has been proven that OFDM is an effective way to increase data rates and simplify the equalization in wireless communications [3].

It is not possible to make reliable data decisions unless a good channel estimate is available. Thus, an efficient and accurate channel estimation procedure is necessary to coherently demodulate received data. Although differential detection could be used to detect the transmitted signal in the absence of channel information, it would result in about 3dB loss in signal to noise ratio (SNR) compared to coherent detection. Several channel estimation algorithms have been reported in the literature [5]-[10]. In these algorithms, however, the channel estimate is continuously updated by transmitting pilot symbols using specified time-frequency lattices. One class of such pilot assisted estimation algorithms adopt an interpolation technique with fixed parameters (two-dimensional (2-D)

[7][8] or one-dimensional (1-D) [6]) to estimate the frequency domain CIR by using channel estimates obtained at the lattices assigned to the pilot tones. Linear, spline and Gaussian filters have all been studied [6]. Another method adopts known channel statistics and channel estimates at pilot tones to estimate CIR in the sense of minimum mean square error (MMSE) [5][9][10]. Shortcomings of these algorithms include (i) a large error floor that may be incurred by a mismatch between the estimated and real CIRs, and (ii) difficulty in obtaining the correlation function of the channel impulse response.

The Expectation-Maximization (EM) algorithm [11] is a technique for finding maximum likelihood estimates of system parameters in a broad range of problems where observed data are incomplete. The EM algorithm consists of two iterative steps: the expectation step and the maximization step. The expectation step is performed with respect to unknown underlying parameters, using the current estimate of the parameters, conditioned upon the incomplete observations. The maximization step then provides a new estimate of the parameters that maximizes the expectation of log likelihood function defined over complete data, conditioned on the most recent observation and the last estimate. These two steps are iterated until the estimated values converge [12].

The main objective of this paper is to investigate use of an EM-based algorithm for signal detection of an OFDM system over a frequency selective channel. For simplicity, we assume the channel is time-invariant during a given OFDM frame period. We leave the time-variant case for a future study.

The rest of the paper is organized as follows. Section II describes the baseband OFDM system model used in the analysis and simulation of the paper. Section III derives the EM-based algorithm to estimate the transmitted signal and CIR. The simplified EM-based algorithm is introduced in section IV. Section V presents computer simulation results to demonstrate the effectiveness of this algorithm. Finally, section VI gives the conclusion.

## II. BASEBAND OFDM SYSTEM MODEL

The schematic diagram of Figure 1 is a baseband equivalent representation of an OFDM system. The input binary data is first fed into a serial to parallel (S/P) converter. Each data stream then modulates the corresponding sub-carrier by MPSK or MQAM. Schemes can vary from one sub-carrier to another in order to achieve the maximum capacity or the minimum bit error rate (BER) under some constraints. In this paper we use, for simplicity, only QPSK or 16QAM in all the sub-carriers. The modulated data symbols, represented by

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complex variables  $X(0), \dots, X(M-1)$ , are then transformed by the inverse fast Fourier transform (IFFT). The output symbols are denoted  $x(0), \dots, x(M-1)$ . In order to avoid ISI, cyclic prefix (CP) symbols, which replicate the end part of the IFFT output symbols, are added in front of each frame. The parallel data are converted back to a serial data stream before being transmitted over the frequency selective channel. The received data  $y(0), \dots, y(M-1)$  corrupted by multipath fading and AWGN are converted back to  $Y(0), \dots, Y(M-1)$  after discarding the prefix, and applying FFT and demodulation.

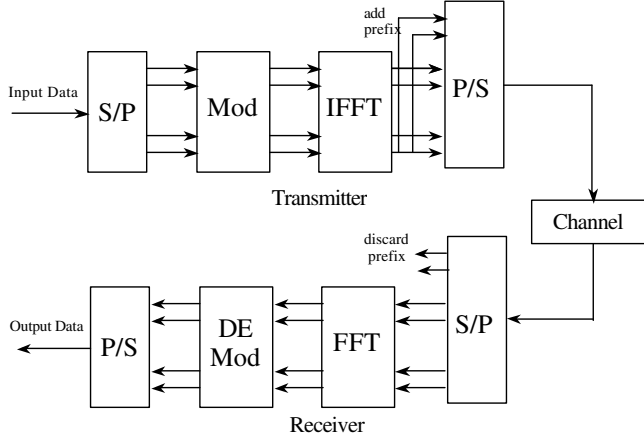


Figure 1: Baseband OFDM system model

The channel model we will adopt in the present paper is a multipath time-invariant fading channel, which can be described by

$$y(k) = \sum_{l=0}^{L-1} h_l x(k-l) + n(k), \quad 0 \leq k \leq M-1, \quad (1)$$

where  $h_l$ 's ( $0 \leq l \leq L-1$ ) are independent complex-valued Rayleigh distributed random variables, and  $n_k$ 's ( $0 \leq k \leq M-1$ ) are independent complex-valued Gaussian random variables with zero mean and variance  $\sigma^2$  for both real and imaginary components.  $L$  is the length of the time-domain CIR.

If we add the cyclic prefix in each OFDM data frame, with its length chosen to be longer than  $L$ , there will be no ISI between OFDM frames. Thus we need to consider only one OFDM frame with  $M$  sub-carriers in analyzing the system performance. After discarding the cyclic prefix and performing an FFT at the receiver, we can obtain the received data frame in the frequency domain:

$$Y(m) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} y(k) e^{-j2\pi \frac{km}{M}}. \quad (2)$$

Substituting (1) into (2), we have

$$Y(m) = X(m)H(m) + N(m), \quad 0 \leq m \leq M-1, \quad (3)$$

where  $H(m)$  is the frequency response of the channel at sub-carrier  $m$ , which can be obtained by

$$H(m) = \sum_{l=0}^{L-1} h_l e^{-j2\pi \frac{ml}{M}}, \quad 0 \leq m \leq M-1, \quad (4)$$

and the set of the transformed noise variables  $N(m)$ ,  $0 \leq m \leq M-1$ , which can be obtained by

$$N(m) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} n(k) e^{-j2\pi \frac{mk}{M}}, \quad 0 \leq m \leq M-1, \quad (5)$$

are i.i.d. complex-valued Gaussian variables and have the same distribution as  $n(k)$ , i.e., with mean zero and variance  $\sigma^2$ .

We adopt  $*$ ,  $T$  and  $H$  to denote complex conjugate, transpose and complex conjugate transpose (Hermite) in the following analysis.

Note that inter-carrier interference (ICI) is also eliminated at the FFT output because of the orthogonality between the sub-carriers.

### III. EM-BASED SIGNAL ESTIMATION ALGORITHM

Our objective is to detect the transmitted signal  $X(m)$ ,  $0 \leq m \leq M-1$  from the observation  $Y(m)$ ,  $0 \leq m \leq M-1$ . In order to reduce bit errors caused by uncertainty in the channel, we apply the following EM-based algorithm to take an average over the unknown CIR, assuming that the probability density function (PDF) of CIR response is known to the receiver.

As stated earlier, the transmitted signal  $X(m)$  is modulated by QPSK or 16QAM. To simplify the expressions, we use  $\underline{H}$ ,  $\underline{h}$ ,  $\underline{X}$ ,  $\underline{Y}$ ,  $\underline{N}$  to denote the vectors of frequency-domain CIR, time-domain CIR, modulated input data, output data and white Gaussian noise respectively, where  $\underline{h} = [h_0, \dots, h_{L-1}]^T$ ,  $\underline{X} = [X(0), \dots, X(M-1)]^T$ ,  $\underline{Y} = [Y(0), \dots, Y(M-1)]^T$ ,  $\underline{N} = [N(0), \dots, N(M-1)]^T$  and  $\underline{H} = \mathbf{W}\underline{h}$ ,  $\mathbf{W}$  is a  $M \times L$  matrix:

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{-j2\pi \frac{1}{M}} & \dots & e^{-j2\pi \frac{L-1}{M}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi \frac{M-1}{M}} & \dots & e^{-j2\pi \frac{(M-1)(L-1)}{M}} \end{bmatrix}_{M \times L}. \quad (6)$$

We also use the notation  $\mathbf{X} = \text{diag}(\underline{X})$ , which denotes a  $M \times M$  matrix with  $X(m)$  as its  $(m, m)$  entry ( $0 \leq m \leq M-1$ ) and zeros elsewhere.

We assume there is no ISI between two successive OFDM symbols due to the assumption that the cyclic prefix is longer than the channel time spread, thus we need to consider only one OFDM symbol at a time. We thus omit the symbol index, and express channel model by

$$\underline{Y} = \mathbf{X}\mathbf{W}\underline{h} + \underline{N}. \quad (7)$$

The EM-based algorithm is used here to obtain an estimate of  $\underline{X}$  that maximizes  $f(\underline{Y}|\underline{X})$  by averaging the logarithm of another likelihood function  $f(\underline{Y}, \underline{h}|\underline{X})$  over the unknown parameters  $\underline{h}$ . The ‘‘incomplete’’ and ‘‘complete’’ data are  $(\underline{Y})$

and  $(\underline{Y}, \underline{h})$ , respectively. Each iterative process  $p = 0, 1, 2, \dots$ , in the EM algorithm for estimating  $\underline{X}$  from  $\underline{Y}$  consists of the following two steps:

$$\mathbf{E} - \text{step} : \quad Q(\underline{X}|\underline{X}_p) = E \left\{ \log f(\underline{Y}, \underline{h}|\underline{X}) | \underline{Y}, \underline{X}_p \right\}, \quad (8)$$

$$\mathbf{M} - \text{step} : \quad \underline{X}_{p+1} = \arg \max_{\underline{X}} Q(\underline{X}|\underline{X}_p), \quad (9)$$

In the E-step at the  $(p+1)^{\text{st}}$  iteration, we compute the expected value of  $\log f(\underline{Y}, \underline{h}|\underline{X})$ , given  $\underline{Y}$  and  $\underline{X}_p$ , the estimate obtained in the  $p^{\text{th}}$  iteration. The M-step of the  $(p+1)^{\text{st}}$  iteration determines the transmitted signal  $\underline{X}_{p+1}$  that maximizes  $Q(\underline{X}|\underline{X}_p)$  given  $\underline{X}_p$ . Equation (8) can be rewritten as

$$Q(\underline{X}|\underline{X}_p) = \int [\log f(\underline{Y}, \underline{h}|\underline{X})] f(\underline{h}|\underline{Y}, \underline{X}_p) d\underline{h}, \quad (10)$$

where the log likelihood function can be express as

$$\log f(\underline{Y}, \underline{h}|\underline{X}) = \log f(\underline{Y}|\underline{h}, \underline{X}) + \log f(\underline{h}|\underline{X}). \quad (11)$$

The conditional PDF  $f(\underline{h}|\underline{Y}, \underline{X}_p)$  is used in (10) to take the conditional expectation over the unknown parameters  $\underline{h}$ . We assume that  $\underline{h}$  and  $\underline{X}$  are independent of each other. This is a reasonable assumption since the CIR does not depend on the transmitted signal in general. Thus, for the purpose of maximization in (9), the Q function of (10) can be replaced by

$$Q(\underline{X}|\underline{X}_p) = \int [\log f(\underline{Y}|\underline{h}, \underline{X})] f(\underline{h}|\underline{Y}, \underline{X}_p) d\underline{h}, \quad (12)$$

The conditional PDF  $f(\underline{h}|\underline{Y}, \underline{X}_p)$  can be calculated by

$$f(\underline{h}|\underline{Y}, \underline{X}_p) = \frac{f(\underline{Y}|\underline{h}, \underline{X}_p) f(\underline{h}|\underline{X}_p)}{f(\underline{Y}|\underline{X}_p)} = \frac{f(\underline{Y}|\underline{h}, \underline{X}_p) f(\underline{h})}{f(\underline{Y}|\underline{X}_p)}.$$

where we use the assumption that  $\underline{h}$  and  $\underline{X}_p$  are independent of each other. Thus, (12) can be further reduced to

$$Q(\underline{X}|\underline{X}_p) = \int [\log f(\underline{Y}|\underline{h}, \underline{X})] f(\underline{Y}|\underline{h}, \underline{X}_p) f(\underline{h}) d\underline{h}, \quad (13)$$

since  $f(\underline{Y}|\underline{X}_p)$  does not depend on  $\underline{X}$ , hence can be discarded in the last expression.

We now compute the above  $Q(\underline{X}|\underline{X}_p)$  for a fading channel with AWGN. The conditional PDFs  $f(\underline{Y}|\underline{h}, \underline{X})$  and  $f(\underline{Y}|\underline{h}, \underline{X}_p)$  take the form

$$f(\underline{Y}|\underline{h}, \underline{X}) = (2\pi\sigma^2)^{-M} \exp \left\{ -\|\underline{Y} - \mathbf{X}\mathbf{W}\underline{h}\|^2 / 2\sigma^2 \right\},$$

$$f(\underline{Y}|\underline{h}, \underline{X}_p) = (2\pi\sigma^2)^{-M} \exp \left\{ -\|\underline{Y} - \mathbf{X}_p\mathbf{W}\underline{h}\|^2 / 2\sigma^2 \right\},$$

where  $\sigma^2$  is the variance of both real and imaginary components of complex-valued Gaussian white noise. The PDF  $f(\underline{h})$  is given by

$$f(\underline{h}) = \frac{1}{(2\pi)^L |\det \boldsymbol{\Sigma}|} \exp \left\{ -\frac{1}{2} (\underline{h} - E\{\underline{h}\})^H \boldsymbol{\Sigma}^{-1} (\underline{h} - E\{\underline{h}\}) \right\}$$

where  $E\{\underline{h}\}$  and  $\boldsymbol{\Sigma}$  are the mean and covariance matrix of the complex-valued CIR vector  $\underline{h}$ . By omitting the constant term and the scaling factor, (12) can be expressed as

$$Q(\underline{X}|\underline{X}_p) = - \int \|\underline{Y} - \mathbf{X}\mathbf{W}\underline{h}\|^2 f(\underline{h}|\underline{Y}, \underline{X}_p) d\underline{h}. \quad (14)$$

And  $f(\underline{h}|\underline{Y}, \underline{X}_p)$  can be represented as

$$f(\underline{h}|\underline{Y}, \underline{X}_p) = K_2 \exp \left\{ -\frac{1}{2} (\underline{h} - \hat{\underline{h}}_p)^H \hat{\boldsymbol{\Sigma}}_p^{-1} (\underline{h} - \hat{\underline{h}}_p) \right\}, \quad (15)$$

where  $K_1$  and  $K_2$  are some constants.  $\hat{\underline{h}}_p$  and  $\hat{\boldsymbol{\Sigma}}_p$  are called the estimated posterior mean and posterior covariance matrix at the  $p^{\text{th}}$  iteration given by

$$\hat{\underline{h}}_p = \hat{\boldsymbol{\Sigma}}_p (\mathbf{W}^H \mathbf{X}_p^H \underline{Y} / \sigma^2 + \boldsymbol{\Sigma}^{-1} E\{\underline{h}\}), \quad (16)$$

$$\hat{\boldsymbol{\Sigma}}_p = (\mathbf{W}^H \mathbf{X}_p^H \mathbf{X}_p \mathbf{W} / \sigma^2 + \boldsymbol{\Sigma}^{-1})^{-1}. \quad (17)$$

Maximization of (14) is equivalent to minimizing the distance

$$\arg \max_{\underline{X}} Q(\underline{X}|\underline{X}_p) = \arg \min_{\underline{X}} E \left\{ \|\underline{Y} - \mathbf{X}\mathbf{W}\underline{h}\|^2 | \underline{Y}, \underline{X}_p \right\}. \quad (18)$$

This minimization can be further simplified as

$$\max_{\underline{X}} E \left\{ \underline{h}^H \underline{F} + \underline{F}^H \underline{h} - \underline{h}^H \mathbf{G} \underline{h} | \underline{Y}, \underline{X}_p \right\}, \quad (19)$$

where

$$\underline{F} = \mathbf{W}^H \mathbf{X}^H \underline{Y}, \quad (20)$$

$$\mathbf{G} = \mathbf{W}^H \mathbf{X}^H \mathbf{X} \mathbf{W}. \quad (21)$$

Since the distribution of random vector  $\underline{Y}$  given  $\underline{h}$  and  $\underline{X}_p$  is Gaussian with mean  $\hat{\underline{h}}_p$  and covariance matrix  $\hat{\boldsymbol{\Sigma}}_p$ , it is easy to compute

$$E \left\{ \underline{h}^H \underline{F} + \underline{F}^H \underline{h} | \underline{Y}, \underline{X}_p \right\} = \hat{\underline{h}}_p^H \underline{F} + \underline{F}^H \hat{\underline{h}}_p. \quad (22)$$

Moreover, all entries of matrix  $\mathbf{G}$  are given in terms of the signal energies, i.e.,  $\|X(0)\|^2, \dots, \|X(M-1)\|^2$ . Thus, we can compute the third part of (19) as

$$E \left\{ \underline{h}^H \mathbf{G} \underline{h} | \underline{Y}, \underline{X}_p \right\} = \sum_{i=0}^{M-1} C_i^2 \|X(i)\|^2, \quad (23)$$

where  $C_i^2$ ,  $0 \leq i \leq M-1$  are some real values dependent on  $\hat{\underline{h}}_p$  and  $\hat{\boldsymbol{\Sigma}}_p$  and can be obtained by the following equation:

$$C_i^2 = \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} e^{j2\pi \frac{(m-n)i}{M}} (\hat{\boldsymbol{\Sigma}}_p(m, n) + \hat{h}_p^*(m) \hat{h}_p(n)). \quad (24)$$

In order to calculate  $Q(\underline{X}|\underline{X}_p)$  completely with respect to  $\underline{X}$ , we write (20) as follows

$$\underline{F} = \mathbf{W}^H \mathbf{Y} \underline{X}^*. \quad (25)$$

Thus, maximizing  $Q(\underline{X}|\underline{X}_p)$  is the same as

$$\max_{\underline{X}} \left\{ \hat{\underline{h}}_p^H \mathbf{W}^H \mathbf{Y} \underline{X}^* + \underline{X}^T \mathbf{Y}^H \mathbf{W} \hat{\underline{h}}_p - \sum_{i=1}^M C_i^2 \|X(i)\|^2 \right\}. \quad (26)$$

Equation (26) can be solved as

$$\tilde{\mathbf{X}}_{p+1} = \arg \max_{\mathbf{X}} Q(\mathbf{X}|\mathbf{X}_p) = \mathbf{C}^{-1} \left( \hat{\mathbf{h}}_p^H \mathbf{W}^H \mathbf{Y} \right)^T, \quad (27)$$

where  $\mathbf{C} = \text{diag}(C_0, \dots, C_{M-1})$ . After quantizing  $\tilde{\mathbf{X}}_{p+1}$  we obtain the  $(p+1)^{\text{st}}$  estimate

$$\mathbf{X}_{p+1} = \text{Quantization}(\tilde{\mathbf{X}}_{p+1}). \quad (28)$$

In each iteration, the updated estimation of channel impulse response  $\hat{\mathbf{h}}_p$  is obtained automatically as a byproduct.

Thus far we have assumed that  $L$ , the number of multipaths, is known. In a real situation, however,  $L$  may not be known. In such a case, we need to perform channel-order detection together with parameter estimation. Alternatively, we may use some upperbound for  $L$ , which may be easier to obtain than trying to estimate an exact value of  $L$ . In an OFDM system we can set  $L$  equal to or less than the length of the cyclic prefix, as we stated earlier that cyclic prefix must be longer than the channel time spread in order to eliminate ISI. Another limitation of our model is that the mean  $E\{\hat{\mathbf{h}}\}$  and the covariance matrix  $\mathbf{\Sigma}$  of time-domain CIR are also assumed to be known. In a practical situation, these channel statistics may not be known.

As is known from the general convergence property of the EM algorithm, there is no guarantee that the iterative steps converge to a global maximum. For a likelihood function with multiple local maxima the convergence point may be one of these local maxima, depending on the initial estimate  $\mathbf{X}_0$ . We, therefore, propose to use pilot symbols distributed at certain locations in the OFDM time-frequency lattice to obtain an appropriate initial value  $\mathbf{X}_0$ , which is more likely to converge to the true maximum point.

#### IV. SIMPLIFIED EM ALGORITHM

In the above analysis, we assumed that the channel statistics (mean and covariance matrix) are known to the receiver. However, as we stated above, exact channel statistics are difficult to obtain in reality. Fortunately, as we examine (16) and (17), we find that when  $\sigma^2$  is small (i.e., signal to noise ratio is high), the contribution of  $\mathbf{\Sigma}$  is so small that we can eliminate it and yet expect a similar performance.

Furthermore, for the MPSK modulated signal, i.e.,  $\|X(m)\|^2 = A$  for all  $m$ , where  $A$  is a positive constant meaning signal energy, and the signal estimation can be performed by using only the phase information. Thus, we can simplify (27) to

$$\tilde{\mathbf{X}}_{p+1} = (\mathbf{Y}^H \mathbf{X}_p \mathbf{W} \mathbf{W}^H \mathbf{Y})^T. \quad (29)$$

It only needs multiplication and addition operations and  $\mathbf{W} \mathbf{W}^H$  can be calculated and stored ahead of time.

#### V. SIMULATION RESULTS

We constructed an OFDM simulate model to demonstrate the validity and effectiveness of the EM-based signal estimation algorithm. The entire channel bandwidth is 400kHz, and is divided into 64 subcarriers (or tones). To make the tones

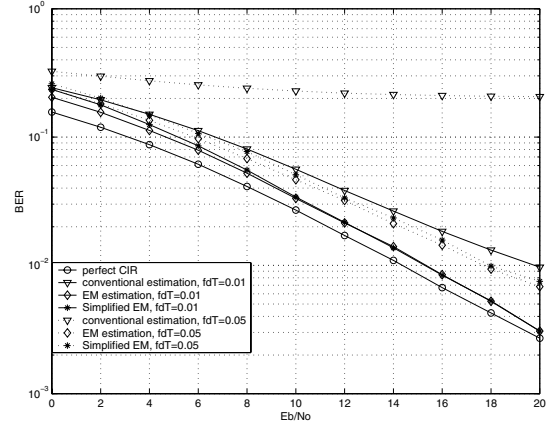


Figure 2: Bit error rate v.s.  $E_b/N_0$

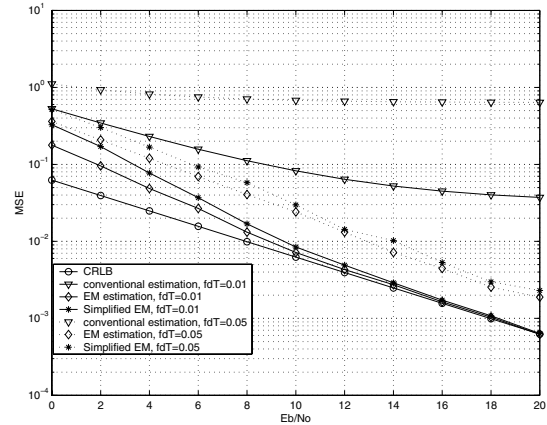


Figure 3: Mean square error v.s.  $E_b/N_0$

orthogonal to each other, the symbol duration is  $160 \mu\text{s}$ . An additional  $20 \mu\text{s}$  cyclic prefix is used to provide protection from ISI and ICI due to channel delay spread. Thus, the total OFDM frame length is  $T_s = 180 \mu\text{s}$  and subchannel symbol rate is 5.56 kbaud. The modulation scheme used in the system is QPSK. One OFDM frame out of 8 OFDM frames has pilot symbols and 8 pilot symbols are inserted into such frame. The simulated system can transmit data at 700 kbits/s. The maximum Doppler frequency is chosen to be 55.6Hz and 277.8Hz, which make  $f_d T_s$  0.01 and 0.05, respectively. The channel impulse response used in the simulation is given by

$$h(n) = \frac{1}{C} \sum_{k=0}^7 e^{-k/2} \alpha_k \delta(n-k),$$

where  $C = \sqrt{\sum_{k=0}^7 e^{-k}}$  is the normalization constant and  $\alpha_k, 0 \leq k \leq 7$  are independent complex-valued Rayleigh distributed random variables with unit energy, which vary in time according to the Doppler frequency. This is a conventional exponential decay multipath channel model.

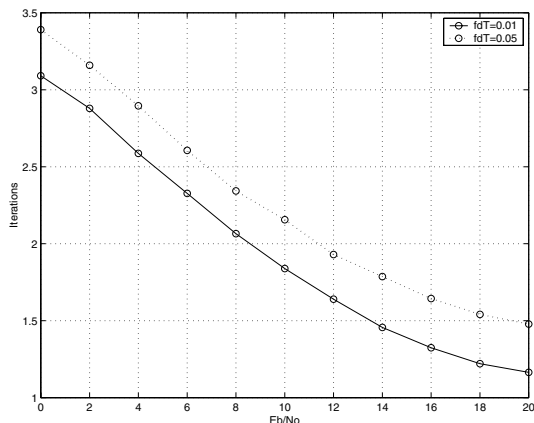


Figure 4: The number of iteration v.s.  $E_b/N_0$

Fig. 2 shows the BER performance of EM-based OFDM signal estimation algorithm with the above two different Doppler frequencies and fig. 3 displays the corresponding MSE. In the conventional channel estimation algorithm, for those OFDM frames containing pilot symbols, the estimate of CIR is obtained by using these 8 equally spaced pilot symbols. For those OFDM frames without pilot symbols, the conventional estimation of CIR comes from the channel estimate of the previous OFDM frame. In the EM-based algorithm we use the estimate of the previous OFDM frame as the initial value for the current OFDM frame if there is no pilot symbols in this frame. From these two figures we can see that the EM-based algorithm can achieve almost as good performance as the ideal case in terms of BER where the channel characteristics are completely known when  $f_d T_s = 0.01$ , i.e., the channel does not change very fast. Furthermore, the MSE of the EM-based channel estimation converges to the Cramer-Rao Lower Bound (CRLB) when  $E_b/N_0$  becomes large. On the other hand, when  $f_d T_s = 0.05$ , the performance of EM-based algorithm cannot achieve that of the ideal case. This is because the pilot symbols contained in the time-frequency grid cannot track the channel variation in such rapid fading. This demonstrates that the performance of the EM-based algorithm depends on the accuracy of the initial estimation. In both cases, the performance gain from the initial estimate is considerably large, especially when  $f_d T_s = 0.05$ . Another interesting result obtained from our simulation is that the performance degradation is quite small when we use the simplified EM-based algorithm that does not use the channel statistics. Thus, this algorithm has a very good robust property.

In Fig. 4, we plot the number of iterations required for the estimates  $\underline{X}_p$  to converge versus  $E_b/N_0$  at the receiver input. We can see that the number of necessary iterations is relatively small in a broad range of  $E_b/N_0$  whether the channel changes slowly enough or not. And the fast channel variation causes only a very small increase in the number of iterations required to converge. This demonstrates that the algorithm can achieve a substantial performance improvement with only a modest increase in the computational complexity.

## VI. CONCLUSION

In this paper we proposed a new EM-based iterative algorithm to efficiently estimate the OFDM transmitted signal that is received from a multipath fading channel with AWGN. The main idea here is to detect the transmitted OFDM frames by averaging over unknown multipath channel parameters. By making use of pilot symbols to obtain the initial estimate, the algorithm can achieve a near-optimal estimate after very few iterations when the channel changes slowly. We also introduce a simplified algorithm without using channel statistics which has almost the same performance as the non-simplified one.

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