EM-Based Channel Estimation for OFDM

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Abstract

Estimating a channel that is subject to frequency selective Rayleigh fading is a challenging problem in an orthogonal frequency division multiplexing (OFDM) system. We propose an EM-based iterative algorithm to efficiently estimate the channel impulse response (CIR) of an OFDM system. This algorithm is capable of improving the channel estimate by making use of pilot tones to obtain the initial estimate for iterative steps. Simulation results show that the bit error rate (BER) can be significantly reduced by this algorithm.

I Introduction

Efficient and accurate channel estimation for OFDM is necessary to coherently demodulate received data. Although differential detection can be used to detect the transmitted signal in the absence of channel information, it results in about 3dB loss in signal to noise ratio (SNR) compared to coherent detection. Several channel estimation algorithms have been reported in the literature. In these algorithms, however, the channel estimate is continuously updated by transmitting pilot symbols using specified time-frequency lattices. One class of such pilot assisted estimation algorithms adopt an interpolation technique wirh fixed parameters (two-dimensional (2-D) [2] or double one-dimensional (1-D) [1]) to estimate the frequency domain CIR by using channel estimates obtained at the lattices assigned to the pilot tones. Linear, spline and Gaussian filters have all been studied [1]. Another method adopts the known channel statistics and channel estimates at pilot tones to estimate CIR in the sense of minimum mean square error (MMSE) [3][4]. Shortcomings of these algorithms include (i) a large error floor that may be incurred by a mismatch between the estimated and the real CIRs, and (ii) the difficulty in obtaining the correlation function of the channel frequency response.

The Expectation-Maximization (EM) algorithm [5] is a technique for finding maximum likelihood estimates of system parameters in a broad range of problems where observed data are incomplete. The main objective of this paper is to investigate use of an EM-based algorithm for channel estimation of an OFDM system.

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II Baseband OFDM System Model

The schematic diagram of Figure 1 is a baseband equivalent representation of an OFDM system. The modulated data symbols, represented by complex variables $X(0), \cdots, X(M-1)$, are then transformed by the inverse fast Fourier transform (IFFT). The output symbols are denoted as $x(0), \cdots, x(M-1)$. The received data $y(0), \cdots, y(M-1)$ corrupted by multipath fading and AWGN are converted back to $Y(0), \cdots, Y(M-1)$ after discarding the prefix, and applying FFT and demodulation.

The channel model we will adopt in the present paper is a multipath time-invariant fading channel, which can

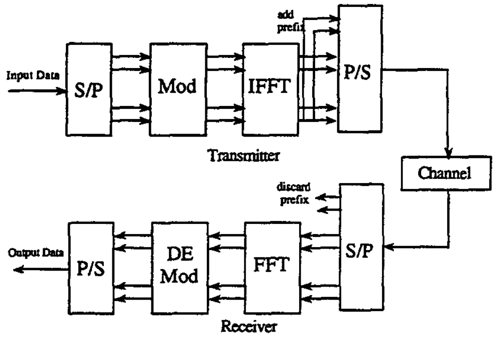

Figure 1: Baseband OFDM system model

be described by

$$y(k) = \sum_{l=0}^{L-1} h_l x(k-l) + n(k), \quad 0 \le k \le M-1, \qquad (1)$$

where h_l's ($0 \le l \le L-1$) are independent complex-valued Rayleigh fading random variables, and n_k's ($0 \le k \le M-1$) are independent complex-valued Gaussian random variables with zero mean and variance σ^2 for both real and imaginary components. L is the length of the time-domain CIR.

The received data in the frequency domain are:

$$Y(m) = X(m)H(m) + N(m), \; 0 \le m \le M-1, \qquad (2)$$

where $H(m)$ is the frequency response of the channel at sub-carrier m, which can be obtained by

$$H(m) = \sum_{l=0}^{L-1} h_l e^{-j2\pi \frac{ml}{M}}, \qquad 0 \le m \le M-1, \qquad (3)$$

and the set of the transformed noise variables $N(m), 0 \le m \le M-1$ are i.i.d. complex-valued Gaussian variables and have the same distribution as $n(k)$.

III EM-Based Channel Estimation Algorithm

As we noted in Section II, we only need to estimate the individual $H(m)'s, 0 \le m \le M-1$, separately, which can considerably reduce the computational complexity. To simplify the expressions, we omit the sub-carrier index m in the following analysis, and write Y, X and H instead of $Y(m)$, $X(m)$ and $H(m)$.

We assume that the transmitted frequency domain signal $X(m)$ is modulated by QPSK or 16QAM with constellation size C (= 4 or 16 respectively). We denote the symbols in the signal constellation by $X_i, 1 \le i \le C$.

The probability density function (PDF) of Y given X and H is given by

$$f(Y|X,H) = \frac{1}{\sqrt{2\pi}\sigma} exp\left\{-\frac{1}{2\sigma^2}||Y - HX||^2\right\}. \qquad (4)$$

By assuming that all these C values are equally likely, and averaging the conditional PDF of (4) over the variable X, we obtain the PDF of Y given H as

$$f(Y|H) = \sum_{i=1}^{C} \frac{1}{\sqrt{2\pi}\sigma C} exp\left\{-\frac{1}{2\sigma^2}||Y - HX_i||^2\right\}. \qquad (5)$$

Suppose that the channel is static over the period of D frames. We define the received signal vector $\underline{Y} = [Y^1, \cdots, Y^D]$ and the transmitted signal vector $\underline{X} = [X^1, \cdots, X^D]$. Then we call $\underline{Y}$ and $(\underline{Y}, \underline{X})$ "incomplete" and "complete" data, respectively, following the terminology of the EM algorithm. Assuming that $Y^1, \cdots, Y^D$ are independent and identically distributed, we can write the PDF of the incomplete data as

$$f(\underline{Y}|H) = \prod_{d=1}^{D} f(Y^d|H). \qquad (6)$$

Thus, the incomplete log-likelihood function is

$$\log f(\underline{Y}|H) = \sum_{d=1}^{D} \log f(Y^d|H), \qquad (7)$$

and the complete data log-likelihood function is given by

$$\log f(\underline{Y}, \underline{X}|H) = \sum_{d=1}^{D} \left\{\log \frac{1}{C} f_{i_d}(Y^d|H)\right\}, \qquad (8)$$

where $\underline{X} = [X_{i_1}^1, \cdots, X_{i_D}^D]$, a relization of transmitted signal, and

$$f_{i_d}(Y^d|H) = \frac{1}{\sqrt{2\pi}\sigma} exp\left\{-\frac{1}{2\sigma^2}||Y^d - HX_{i_d}^d||^2\right\}, \qquad (9)$$

and $i_d \in \{1, 2, \cdots, C\}, 1 \le d \le D$, denote the index number of in the signal constellation.

Each iterative process $p = 0, 1, 2, \cdots$, in the EM algorithm for estimating H from $\underline{Y}$ consists of the following two steps:

E-step: $Q\left(H|H^{(p)}\right) = E\left\{\log f(\underline{Y}, \underline{X}|H)|\underline{Y}, H^{(p)}\right\}$,
M-step: $H^{(p+1)} = \arg\max_{H} Q\left(H|H^{(p)}\right)$, where

$$Q\left(H|H^{(p)}\right) = \sum_{i=1}^{C}\sum_{d=1}^{D} \log\left\{\frac{1}{C}f_i(Y^d|H)\right\}\frac{f_i(Y^d|H^{(p)})}{Cf(Y^d|H^{(p)})}.$$

The conditional PDFs $f_i(Y^d|H^{(p)})$ and $f(Y^d|H^{(p)})$ can be calculated from (9) and (5).

After discarding some constants and setting the derivative of (10) to zero corresponding to H, we obtain the following equation

$$\begin{aligned} \tilde{H}^{(p+1)} &= \left[\sum_{i=1}^{C}\sum_{d=1}^{D} X_i X_i^H \frac{f_i(Y^d|H^{(p)})}{f(Y^d|H^{(p)})}\right]^{-1} \\ &\times \left[\sum_{i=1}^{C}\sum_{d=1}^{D} Y^d X_i^H \frac{f_i(Y^d|H^{(p)})}{f(Y^d|H^{(p)})}\right]. \qquad (10) \end{aligned}$$

The channel estimates of the form (10) obtained for the M sub-channels, which we denote $\tilde{H}^{(p+1)}, 0 \le m \le M-1$, must be passed through a low pass filter (LPF) in order eliminate the noise from paths that do not exist. The LPF operation can be simply realized by applying the IFFT followed by the FFT, as schematically shown in Figure 2. The values $h_l^{(p+1)}, L \le l \le M-1$ obtained by IFFT must be set to zeros before performing FFT.

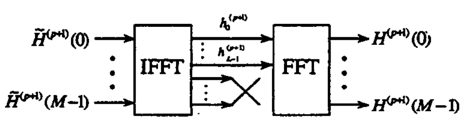

Figure 2: Low Pass Filter

We use pilot symbols distributed at certain locations in the OFDM time-frequency grid to find an appropriate initial value $H^{(0)}$. The iteration should be terminated as soon as the difference between $H^{(p+1)}$ and $H^{(p)}$ is sufficiently small.

IV Simulation Results

To demonstrate the validity and effectiveness of the EM-based channel estimation algorithm, we built a simulation model of an OFDM system with 64 sub-carriers. An OFDM frame consists of 71 symbols including 7 symbols for cyclic prefix ($N_c = 7$) and 4 symbols for pilots ($N_p = 4$). The transmission efficiency is therefore 84.5% (= 60/71). The modulation scheme used in this simulation is QPSK. The following three multipath channel models with L=2, 3 and 5, are considered.

$$\begin{aligned} h_1(n) &= 0.8\alpha_0\delta(n) + 0.6\alpha_1\delta(n-1) \\ h_2(n) &= 0.3\alpha_0\delta(n) + 0.813\alpha_1\delta(n-1) + 0.5\alpha_2\delta(n-2) \\ h_3(n) &= 0.41\alpha_0\delta(n) + 0.3\alpha_1\delta(n-1) + 0.7\alpha_2\delta(n-2) \\ &+ 0.5\alpha_3\delta(n-3) + 0.2\alpha_4\delta(n-4) \end{aligned}$$

where $\alpha_i, 0 \le i \le 4$ are independent complex-valued Rayleigh distributed random variables with unit energy. We select $D = 1$. Figures 3, 4 and 5 show the system performance in terms of the symbol error rate versus E_b/N_0.

From Figures 3 and 4, we can see that when the number of pilot symbols per frame is greater than the channel spread, allowing us to obtain a very good initial estimate, the EM-based algorithm can achieve performance close to the case where the channel characteristic is completely known. On the other hand, Figure 5 shows that when the number of pilot symbols per frame is less than the channel spread, the initial estimate may not be accurate enough. In this case, the EM-based algorithm cannot achieve the global maximum. However, it can still improve the overall performance.

In Figure 6, where we plot the number of iterations required for the estimates $H^{(p)}$ to converge versus different values of the input E_b/N_0. We see that the number of necessary iterations decreases rapidly as E_b/N_0 increases. When $E_b/N_0 = 20dB$, for instance, only three or four steps are needed to achieve the convergence in the two-path and three-path cases.

V Conclusion

In this paper we proposed an EM-based iterative algorithm to efficiently estimate the OFDM channel impulse response (CIR) for multipath fading channels with additive white Gaussian noise (AWGN). By making use of pilot tones to obtain the initial estimate, the algorithm can achieve a near-optimal estimate after a few iterations. It has been shown in our simulation that when the number of pilot symbols is larger than the channel spread, the algorithm can always achieve the global maximum. Our algorithm can easily be extended to estimate multiple input/multiple output (MIMO) channels.

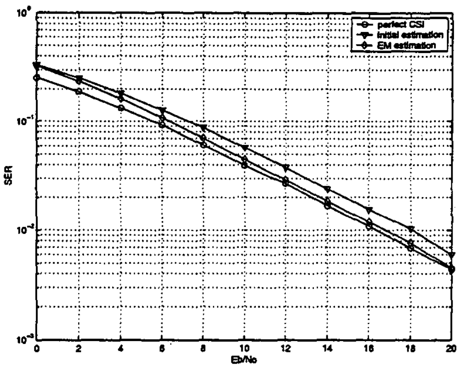

Figure 3: Symbol error rate v.s. E_b/N_0 in the two-path channel model

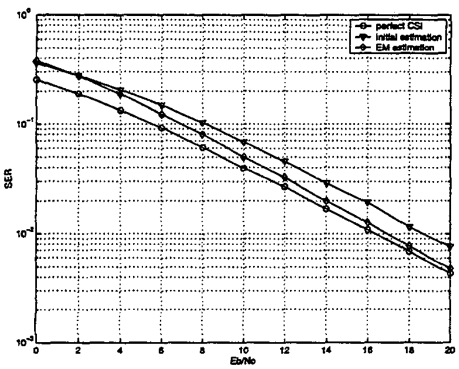

Figure 4: Symbol error rate v.s. E_b/N_0 in the three-path channel model

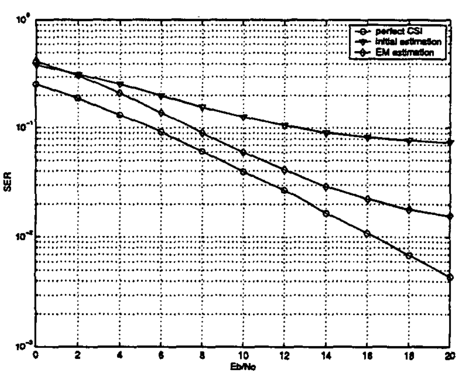

Figure 5: Symbol error rate v.s. E_b/N_0 in the five-path channel model

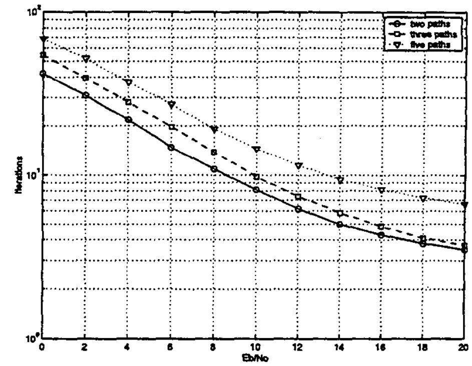

Figure 6: The number of iterations required for convergence v.s. E_b/N_0

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