Effects of Multipath Interference on an FM Data Subcarrier

Linda Zeger *Pei Chen and Hisashi Kobayashi
Department of Electrical Engineering
School of Engineering and Applied Science
Princeton University

Abstract

There is a renewed interest in utilizing the upper portion of the baseband spectrum of an FM broadcast channel for digital data service. This part of the baseband spectrum was previously unused by the audio signal. In the presence of multipath interference, the instantaneous frequency of the received FM signal is a nonlinear combination of the instantaneous frequency of the transmitted signal and its time delayed versions. Due to this nonlinearity, the audio and data signals are no longer confined to separate parts of the baseband spectrum. Correct reception of the data signal is then limited by the effects of multipath interference from the audio signals. In this paper we formulate the multipath interference problem, and present some preliminary analysis and simulation results.

1 Introduction

A number of companies have recently begun to transmit data by using the existing FM systems that broadcast audio signals. Examples of such datacasting services are: RDS (radio data service) used and standardized in Europe (and its US version called RDBS, although not widely used yet); DARC (DAta Radio Channel) system developed by NHK [1] and now serviced by some FM broadcasters in Japan; and HSDS (High Speed Data Service) [2], a paging service provided by Seiko Communications.

Audio signals occupy the portion of the baseband spectrum from 0 to 53 kHz, while frequencies from 53 kHz to 100kHz were previously unutilized. This upper portion of the spectrum can be used to transmit digital data (as is seen for example by the baseband spectrum displayed in Figure 1), so that both the audio signal $A(t)$ and the data signal $D(t)$ modulate the FM carrier of frequency f_c. The instantaneous frequency of the transmitted signal is then

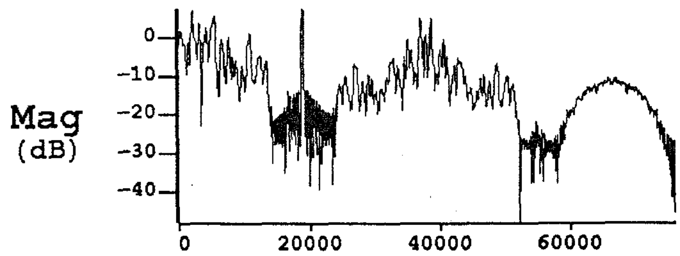


Figure 1: Power Spectral Density of Transmitted Audio and Data Signals

$$\omega(t) = 2\pi f_c + 2\pi f_d\,[\mathrm{A}(t) + \mathrm{D}(t)] = 2\pi f_c + 2\pi f_d S(t), \quad (1)$$

where f_d is the frequency deviation constant, and $S(t)$ is the total modulating signal. The transmitted signal is then given by

$$X(t) = A\cos(\int_0^t \omega(u)du), \quad (2)$$

or

$$X(t) = A\cos(\omega_c t + \theta_s(t)) \quad (3)$$

where

$$\theta_s(t) = \omega_d \int_0^t S(u)\,du = 2\pi f_d \int_0^t S(u)\,du \quad (4)$$

denotes the phase of the FM signal.

In the absence of multipath interference, the audio signal $A(t)$ and the data signal $D(t)$ could be separated by filtering the instantaneous frequency in the baseband after demodulation, because the audio and data signals occupy separate frequency ranges. However, when there is multipath interference, the instantaneous frequency of the received signal is a nonlinear combination of the instantaneous frequency of the transmitted signal and its time delayed versions, as shown below. The audio and data signals are no longer confined to separate parts of the baseband spectrum; hence, the multipath

*Currently with Lucent Technologies

0-7803-3905-3/97/$10.00

channel results in interference between the audio and data signals.

The maximum amplitudes of the audio and data signals are regulated according to:

$$f_d \mid \mathrm{A}(t) \mid \;\leq\; 75\text{kHz}. \quad (5)$$
$$f_d \mid \mathrm{D}(t) \mid \;\leq\; 7.5\text{kHz}. \quad (6)$$

so that typically the audio signal peak power is approximately one hundred times that of the data signal. Thus it appears that multipath interference could potentially cause the audio signal to significantly degrade the data signal.

In this paper, we analyze the effects of multipath interference on the FM data subcarrier. We begin with a simple two path model, and then generalize the two path model to include the effects of additive noise, numerous paths, and phase differences associated with each path. For the two path model, we discuss the interference effects of the data with itself as well as the interference of the audio signal with the data, and derive analytic results when the amplitude of the secondary path is relatively attenuated and the delay times are approximately 1 μsec or less. We present expressions for the received instantaneous frequency, as well as its power spectral density in the frequency range of the transmitted data in this case. It is shown that for the small attenuation coefficients and delays considered here, the audio signal does not significantly degrade the data. Simulations are used to study multipath effects for longer delay times and larger attenuation coefficients.

2 Two Path Model

We first investigate multipath interference effects on frequency modulation by considering a simple two path multipath model. The received signal $Y(t)$ is given by

$$Y(t) = X(t) + \alpha X(t-\tau), \quad (7)$$

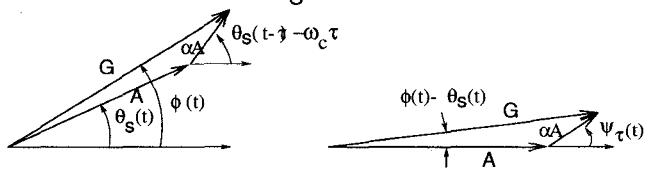
where α and τ are the amplitude and time delay of the second path relative to the primary path. Substituting (3) for $X(t)$, we obtain

$$\begin{aligned} Y(t) &= A\cos(\omega_c t + \theta_s(t)) \\ &\quad +\alpha A\cos(\omega_c t + \theta_s(t-\tau) - \omega_c\tau) \\ &= G\cos(\omega_c t + \phi(t)) \end{aligned} \quad (8)$$

in terms of the envelope G and phase $\phi(t)$ of the received signal. The phase $\phi(t)$ is then given by

$$\begin{aligned} \phi(t) &= \theta_s(t) + \tan^{-1}\frac{\alpha\sin(\psi_\tau(t))}{1+\alpha\cos(\psi_\tau(t))} \\ &= \theta_s(t) + \xi(t) \end{aligned} \quad (9)$$

Figure 2:



(a) (b)

where

$$\begin{aligned} \psi_\tau(t) &= \theta_s(t-\tau) - \theta_s(t) - \omega_c\tau \\ &= \int_t^{t-\tau} [\omega_c + \omega_d S(u)]\,du \\ &= \int_t^{t-\tau} \omega(u)\,du, \end{aligned} \quad (10)$$

where $\omega(t)$ is the instantaneous frequency of the transmitted signal $X(t)$. These relationships can also be seen graphically, as in Figure 2a or 2b.

The FM discriminator output is the instantaneous frequency of the received signal:

$$\omega_r(t) = \omega_c + \dot{\phi}(t) = \omega_c + \omega_d S(t) + \dot{\xi}(t). \quad (11)$$

The term $\dot{\xi}(t)$ is the part of the FM discriminator output due to multipath interference, which is given by

$$\dot{\xi}(t) = \frac{\alpha\,(\alpha + \cos\psi_\tau(t))\,\dot{\psi}_\tau(t)}{1 + 2\alpha\cos\psi_\tau(t) + \alpha^2} = \eta(t)\dot{\psi}_\tau(t), \quad (12)$$

where the weight $\eta(t)$ is defined by

$$\eta(t) = \frac{\alpha\,(\alpha + \cos\psi_\tau(t))}{1 + 2\alpha\cos\psi_\tau(t) + \alpha^2}. \quad (13)$$

Thus the instantaneous frequency of the received signal can be written as

$$\omega_r(t) = [1 - \eta(t)]\,\omega(t) + \eta(t)\omega(t-\tau). \quad (14)$$

Since the phase (10) depends on $\omega(t)$, the weight $\eta(t)$ also depends on $\omega(t)$. Thus equation (14) shows that the instantaneous frequency of the received signal is a nonlinear combination of the transmitted instantaneous frequency and its time delayed version.

3 Two Path Model with Additive Noise

We next consider the case in which the channel contributes both multipath interference *and* additive narrowband noise $N(t)$. In this case the received FM signal is

$$Y(t) = X(t) + \alpha X(t-\tau) + N(t), \quad (15)$$

where α and τ are again the attenuation and delay of the secondary path signal, and the IF noise $N(t)$ is given by

$$\begin{aligned} N(t) &= N_I(t)\cos\omega_c t - N_Q(t)\sin\omega_c t && (16)\\ &= V_n(t)\cos(\omega_c t + \theta_n(t)), && (17)\end{aligned}$$

where $V_n(t)$ is called the noise envelope, and $\theta_n(t)$ is the noise phase. In the same manner as was done in Section 2, we can rewrite the received signal as

$$\begin{aligned} Y(t) &= A\cos(\omega_c t + \theta_s(t)) + \alpha A\cos(\omega_c t + \theta_s(t-\tau) - \omega_c\tau)\\ &\quad + V_n(t)\cos(\omega_c t + \theta_n(t)) && (18)\\ &= G(t)\cos(\omega_c t + \phi(t)) && (19)\end{aligned}$$

where now

$$\begin{aligned} \phi(t) &= \theta_s(t) + \xi(t) && (20)\\ \tan\xi(t) &= \frac{\alpha A\sin\psi_\tau(t) + V_n\sin\Delta\theta(t)}{A + \alpha A\cos\psi_\tau(t) + V_n\cos\Delta\theta(t)}, && (21)\end{aligned}$$

and $\Delta\theta(t)$ is defined by

$$\Delta\theta(t) = \theta_n(t) - \theta_s(t). \qquad (22)$$

The phase difference $\psi_\tau(t)$ is due to the multipath interference, and is given by equations (4) and (10), and $\Delta\theta(t)$ is due to noise.

We shall write $\xi, V_n, \psi, \Delta\theta$ instead of $\xi(t), V_n(t), \psi_\tau(t)$ and $\Delta\theta(t)$. Then the part of the FM discriminator output $\dot\phi$ due to noise and multipath interference is given by

$$\dot\xi = \frac{D}{N} \qquad (23)$$

where

$$\begin{aligned} D &= A^2\alpha\dot\psi(\alpha + \cos\psi) + A\alpha[V_n(\dot\psi + \Delta\dot\theta)\cos(\psi - \Delta\theta)\\ &\quad - \dot V_n\sin(\psi - \Delta\theta)] + A\dot N_q + V_n^2\Delta\dot\theta && (24)\\ N &= A^2(1 + 2\alpha\cos\psi + \alpha^2)\\ &\quad + 2AV_n[\cos\Delta\theta + \alpha\cos(\psi - \Delta\theta)] + V_n^2. && (25)\end{aligned}$$

and N_q is defined by

$$N_q = V_n\sin\Delta\theta. \qquad (26)$$

For high FM signal to noise ratios, that is when $A \gg V_n$:

$$\dot\xi \approx \frac{\alpha(\alpha + \cos\psi)\dot\psi}{1 + 2\alpha\cos\psi + \alpha^2} = \eta(t)\dot\psi \qquad (27)$$

where

$$\begin{aligned} \eta(t) &= \frac{\alpha(\alpha + \cos\psi)}{1 + 2\alpha\cos\psi + \alpha^2}. && (28)\\ \dot\psi &= \dot\theta_s(t-\tau) - \dot\theta_s(t)\\ &= 2\pi f_d[S(t-\tau) - S(t)]. && (29)\end{aligned}$$

In the limit of $A \gg V_n$, the FM discriminator output is

$$\begin{aligned} \dot\phi(t) &= \dot\theta_s(t) + \dot\xi(t)\\ &\approx 2\pi f_d[(1 - \eta(t))S(t) + \eta(t)S(t-\tau)]. && (30)\end{aligned}$$

The discriminator output is the instantaneous frequency of the received signal, which can be written in this approximation as

$$\omega_r(t) \approx (1 - \eta(t))\omega(t) + \eta(t)\omega(t-\tau). \qquad (31)$$

Note that when the FM signal is large relative to the noise, the instantaneous frequency derived in (31) and (28) is the same as that derived previously in equations (14) and (13) for the case of a multipath channel with no noise.

4 Generalized Multipath Model

Equations (15)-(31) can be generalized to the case where there are more than two multipath reflections, and each reflected path has its own phase change relative to the transmitted signal. For simplicity we illustrate this generalization for the case without additive noise, which can also be seen as an extension of equations (9) to (14). We consider the more general multipath model

$$\begin{aligned} Y(t) &= X(t) + \alpha_1 X(t-\tau_1,\theta_1) + \alpha_2 X(t-\tau_2,\theta_2)\\ &\quad + \ldots + \alpha_M X(t-\tau_M,\theta_M), && (32)\end{aligned}$$

where there are a total of M reflected paths. The phase, attenuation, and delay of the i^{th} reflected path are denoted by θ_i, α_i, and τ_i respectively. In analogy to equations (20) and (21), the phase $\phi(t)$ of the received signal $Y(t)$ can be expressed as

$$\phi(t) = \theta_s(t) + \xi(t), \qquad (33)$$

where

$$\tan\xi(t) = \frac{\sum_{i=1}^{M}\alpha_i\sin(\psi_i(t))}{1 + \sum_{i=1}^{M}\alpha_i\cos(\psi_i(t))}. \qquad (34)$$

The phase $\psi_i(t)$, a generalization of (10), is given by

$$\begin{aligned} \psi_i(t) &= \theta_s(t-\tau_i) - \theta_s(t) - \omega_c\tau_i + \theta_i && (35)\\ &= \int_t^{t-\tau_i}[\omega_c + \omega_d S(u)]du + \theta_i. && (36)\end{aligned}$$

Furthermore, the interference term in the discriminator output is given by

$$\dot\xi = \sum_{i=1}^{M}\eta_i(t)\dot\psi_i(t), \qquad (37)$$

where

$$\eta_i(t) = \frac{D_i}{N_i} \quad (38)$$

$$D_i = \alpha_i \cos\psi_i + \alpha_i \sum_{j=1}^{M} \alpha_j \cos(\psi_i - \psi_j) \quad (39)$$

$$N_i = 1 + 2\sum_{j=1}^{M} \alpha_j \cos\psi_j + \sum_{j=1}^{M}\sum_{k=1}^{M} \alpha_j\alpha_k \cos(\psi_j - \psi_k) \quad (40)$$

$$\dot{\psi}_i(t) = \dot{\theta}_s(t-\tau_i) - \dot{\theta}_s(t) = 2\pi f_d[S(t-\tau_i) - S(t)] \quad (41)$$

Equations (37) and (38) are seen to be natural extensions of equations (12) and (13) of the simple two path model.

Finally, we find that the generalization of (30) for the discriminator output is given by

$$\dot{\phi}(t) = 2\pi f_d \left[\left(1 - \sum_{i=1}^{M} \eta_i(t)\right) S(t) + \sum_{i=1}^{M} \eta_i(t) S(t-\tau_i)\right] \quad (42)$$

5 Mild Multipath Conditions

By using simulations, we can observe the power spectral density of the received signal in the digital frequency range, denoted (f_D^-, f_D^+). Using the two path model of equation (7), in the limit of small α and small τ we can obtain an analytic expression for the power spectral density. We derive this expression by first noting that for small α and small τ the Taylor series approximations allow the received instantaneous frequency in the digital frequency range (f_D^-, f_D^+) to be written, after using trigonometric identities, as:

$$\begin{aligned}\omega_r(t) = & \int_{f_D^-}^{f_D^+} [Re\{D(f)\}\cos(2\pi f t) \\ & -Im\{D(f)\}\sin(2\pi f t)]\,df \\ & + 2\alpha\cos(2\pi f_c\tau)\int_{f_D^-}^{f_D^+} \sin(\pi f\tau)\,\times \\ & [Re\{D(f)\}\sin(2\pi f(t-\frac{\tau}{2})) \\ & +Im\{D(f)\}\cos(2\pi f(t-\frac{\tau}{2}))]\,df \\ & - 2\alpha\sin(2\pi f_c\tau)\int_0^{f_A^+}\int_0^{f_A^+} \sin(\pi f\tau)\sin(\pi f'\tau)\frac{1}{2\pi f'} \\ & [Re\{A(f)\}Re\{A(f')\} - Im\{A(f)\}Im\{A(f')\}] \\ & \times\sin\left(2\pi(f+f')(t-\frac{\tau}{2})\right)\,df\,df' + \ldots \quad (43)\end{aligned}$$

where $(0, f_A^+)$ represents the spectral band of audio.

Equation (43) includes terms up to order α. The first term in the equation is the transmitted digital data signal, while the remaining terms are due to multipath interference. The last term is due to the audio signal's interference in data frequency range (f_D^-, f_D^+). Although the transmitted audio signal is confined to the $(0,\ f_A^+)$ region, the nonlinearity in the received signal due to multipath, as seen in equation (14), produces these audio interference terms in (f_D^-, f_D^+). There are additional terms linear in α which depend on both $A(f)$ and $D(f)$.

We next compute the power spectral density $\Psi_r(f)$ of the received signal $\omega_r(t)$ in the data frequency range (f_D^-, f_D^+), when both multipath parameters α and τ are small. We assume independent Gaussian random processes for the components $V_{L-R}(t)$ and $V_{L+R}(t)$ of the audio signal. Furthermore, we assume A(t) is independent of D(t). We denote

$$\begin{aligned}\Psi_D(f) &= \text{power spectral density of D(t)} \\ \Psi_A(f) &= \text{power spectral density of A(t)}\end{aligned}$$

Using equation (43) and including additional terms to yield the power spectral density with terms up to order α^2, we derive

$$\begin{aligned}\Psi_r(f) = & \Psi_D(f)\left[1 - \frac{\alpha}{2}\cos(2\pi f_c\tau)\sin^2\pi f\tau\right] \\ & + \alpha^2\frac{\sin^2 2\pi f_c\tau}{16\pi^2}\int_0^{f_A^+} \Psi_A(f)\Psi_A(f-f_1) \\ & \times\sin^2(\pi f_1\tau)\sin^2(\pi(f-f_1)\tau) \\ & \times\frac{1}{f_1}\left(\frac{1}{f_1} + \frac{1}{f-f_1}\right)df_1 + \ldots \quad (44)\end{aligned}$$

The first term $\Psi_D(f)$ is the power spectral density of the transmitted data signal. The next term, the correction linear in α, is due to multipath interference of the data signal with itself. The remaining terms, which are quadratic in α, originate from the audio signal, which when the FM signal is passed through the multipath channel, contribute to the data frequency range (f_D^-, f_D^+) of the received demodulated signal. If values of $\tau \leq 1\mu$sec and $\alpha \leq .3$, along with $f_A^+ = 53$ kHz, are substituted into equation (44), then in the range $f_D^- = 57$ kHz to $f_D^+ = 76$ kHz for example, the multipath terms contribute only a small amount to the total power spectral density; hence, multipath effects are not significant here when the multipath parameters are small enough. For larger values of α and τ, interference effects can be investigated with simulations.

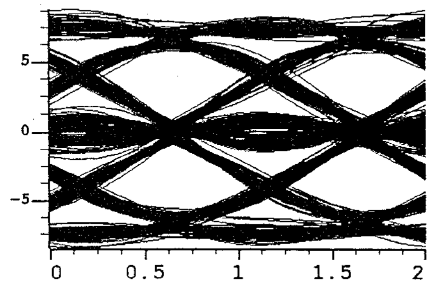


Figure 3: τ = 1 microsecond, α = 0.1

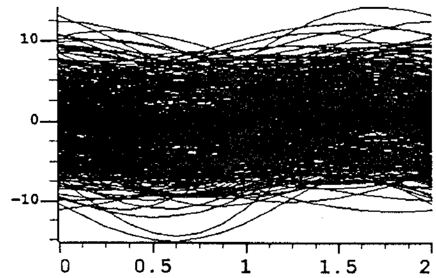


Figure 4: τ = 2.8 microseconds, α = 0.75

6 Simulation Results

Using our simulation system, we further studied multipath interference effects by comparing eye disgrams and power spectral density plots of the baseband signal at the transmitter with those at the receiver following various multipath channels. Our simulation results show that:

- For small delay τ and amplitude α, the eye of the received data signal is quite clear (Figure 3), and the power spectral density of the received signal is almost identical to that of the transmitted signal, which agrees with the analytical result.
- For small α, the interference grows with delay.
- When α becomes large, for small τ the degree of interference oscillates with delay, and when τ is large, the interference becomes very strong. Figure 4 shows an intermediate value of τ at which the interference significantly degrades the data signal. When τ is small compared to the time over which the frequency deviation changes significantly, degradation by multipath interference oscillates with τ according to when the transmitted and time delayed signals interfere constructively or destructively, and hence this oscillation depends on f_c.
- For small delay, the interference is not strong for a wide range of α. But for large delay, the interference goes stronger as α increases.

7 Conclusion and Future Studies

We developed both analytical techniques and simulation models to study the adverse effects of multipath interference. Our study can be extended in following aspects:

- Use Rayleigh fading and Doppler-shift models to understand how these fading channels affect the system performance.
- Study optimal coding and modulation schemes that can reduce the effects of multipath interference and fading effects.

We also believe that the multipath interference analysis technique we have developed may be applicable to analyze other systems, such as GSM (the digital cellular system developed in Europe but is being marketed in the U.S., Australia and many Asian countries).

Acknowledgments

This work has been supported, in part, by the National Science Foundation, Seiko Communications, and the Ogasawara Foundation for the Promotion of Science and Engineering.

References

1. Kuroda, T., M. Takada, T. Isobe, and O. Yamada, "Transmission Scheme of High Capacity FM Multiplex Broadcasting System" *IEEE Trans. Broadcasting* Vol. 42, No. 3, pp. 245-250, Sept. 1996.
2. Gaskill, G. "Seiko High Speed Subcarrier Data System, HSDS: White Paper," Seiko Telecommunication Systems, Inc., Sept. 1993.
3. Schwartz, M., W. R. Bennet, and S. Stein, *Communication Systems and Techniques*, McGraw-Hill, Inc. 1966.