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Improved Low-Complexity Turbo Decoding for HSDPA Applications

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Abstract — **We investigate efficient use of the scaling approach in improving the performance of sub-optimum low-complexity soft output turbo decoding algorithms, such as SOVA (soft output Viterbi algorithm). Simulations using the WCDMA/HSDPA parameters, specified by the 3GPP standard, demonstrate that incorporating a simple scaling factor during the calculation of extrinsic information, coding gains of about 0.5 dB and 0.7 dB can be obtained at BER (bit error rate) of 10^{-3} and FER (frame error rate) of 7×10^{-3}, respectively, compared to the conventional SOVA (without scaling). The performance of the improved SOVA is much closer to that of the optimum Log-MAP decoding than SOVA.**

I. INTRODUCTION

Low-complexity soft output algorithms, used for iterative decoding of turbo codes, have drawn a great deal of attention in recent years because of their ability to provide trade-offs between error performance and implementation complexity. Among various low-complexity algorithms known today, SOVA (soft-output Viterbi algorithm) [1] has been most favorably received. This decoding algorithm, which involves essentially the same operations as the Viterbi Algorithm (VA) only with additional real value additions and storage for soft reliability information, is significantly less complex than the MAP (maximum *a posteriori* probability) algorithm [2], an optimum decoding algorithm for turbo codes but requires significantly greater computations. However, the original SOVA suffers considerable performance degradation compared to MAP. There are some logarithmic MAP like algorithms, such as Log-MAP and Max-Log-MAP [3], which are based on the same decoding principle as the conventional MAP but operate in the logarithmic domain. Although such algorithms can solve the complex nonlinear numerical representation problems caused by MAP and outperform SOVA, they still require greater computational complexity compared to SOVA.

It has recently been reported that scaling extrinsic information, which is soft reliability information exchanged between constituent decoders of a turbo code, can improve performance of some sub-optimum turbo decoding algorithms [4, 5]. An explanation of such effect is that for a bad channel, the soft outputs of these sub-optimal algorithms are severely distorted due to high correlation between extrinsic and systematic information. Using a scaling technique incorporated with the extrinsic information calculation can correct the distortions and so improve decoding quality. It provides an attractive approach to achieve near-optimum decoding with lower complexity.

In this paper, we investigate the scaling effect in improving SOVA decoding of a turbo code adopted in the 3rd generation HSDPA (high-speed downlink packet access) system. As is well known, turbo codes have been found very effective in mobile wireless communications, where SNR is generally low, because of their outstanding error correcting capability. And HSDPA is currently emerging as an extension of WCDMA (wideband CDMA), specified by 3GPP (the 3rd generation partnership project), with aim at providing even higher data rates (about 8-10 Mbit/s) for future packet-based multimedia services. We have searched for good scaling factors suitable for a WCDMA/HSDPA environment and estimated the BER (bit error rate) and FER (frame error rate) of the improved SOVA as well as the Log-MAP algorithm. Simulation results show that by using a simple scaling factor within the extrinsic information calculation of SOVA decoding, coding gains of about 0.5 dB and 0.7 dB can be obtained at BER (bit error rate) of 10^{-3} and FER (frame error rate) of 7×10^{-3}, respectively, over the conventional SOVA (without scaling). The performance of the improved SOVA approaches much closer to that of the optimum Log-MAP decoding than SOVA.

II. SYSTEM MODEL

Consider the HSDPA transmission system model of Fig. 1. A rate-1/4 turbo code is used in the system. Two schemes are proposed to obtain a 1/4 code rate in the 3GPP/HSDPA standard [6]: one is by repetition from a rate-1/3 code; the other by puncturing a rate-1/6 code. Since the repetition scheme allows a simpler implementation, it is of more practical interest, and so we adopt this scheme in our study. In this scheme, the turbo encoder is constructed by parallel concatenation of two identical RSC (recursive systematic convolutional) constituent encoders with a turbo internal interleaver in-between. Each RSC encoder has a rate-1/2 with constraint length $K = 4$ (i.e., memory $\nu = 3$), parity polynomial $g_1(D) = 1 + D^2 + D^3$ and feedback polynomial $g_0(D) = 1 + D + D^3$, as shown in Fig. 2. For each input information sequence block $\mathbf{u} = \{u_1, u_2, \ldots, u_N\}$ of length N, $u_k \in \{0, 1\}$ for $k = 1, 2, \ldots, N$, RSC1 operates directly on it and produces the first parity sequence $\mathbf{Y} = \{Y_1, Y_2, ..., Y_N\}$; RSC2 operates on the interleaved version of $\mathbf{u}$ and produces the second parity sequence $\mathbf{Y}' = \{Y'_1, Y'_2, \ldots, Y'_N\}$. The resultant turbo-coded sequence $\mathbf{C} = \{C_1, C_2, \ldots, C_N\}$ is the sum of the three components $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Y}'$, i.e., $\mathbf{C} = (\mathbf{u}, \mathbf{Y}, \mathbf{Y}')$. Then repeat both parity bits and systematic bits equally in every fourth bit. The overall output sequence after the rate matching is $\{u_1, Y_1, Y'_1, u_1, u_2, Y_2, Y'_2, Y_2, u_3, Y_3, Y'_3, u_3, u_4, Y_4, Y'_4, Y'_4, \ldots\}$. This sequence is transmitted over a fading channel, for which channel interleaving, QPSK modulation and spreading are employed. The received sequence is fed into the turbo decoder, denoted $\mathbf{R} = \{R_1, R_2, \ldots, R_N\}$, where $\mathbf{R} = (\mathbf{x}, \mathbf{y}, \mathbf{y}')$ and $R_k = (x_k, y_k, y'_k)$ is the noise corrupted version of C_k at time k (assume sufficient channel interleaving), i.e.,

$$x_k = a_k(2u_k - 1) + p_k \tag{1}$$

$$y_k = a_k(2Y_k - 1) + q_k \quad \text{or} \quad y'_k = a_k(2Y'_k - 1) + q_k \tag{2}$$

where a_k is the instantaneous fading amplitude with a Rayleigh pdf. $p_A(a_k) = \left(a_k/\sigma_A^2\right)\exp\left(-a_k^2/2\sigma_A^2\right)$ for $a_k > 0$, and p_k and q_k are two independent Gaussian noise variables, both with zero mean and variance σ^2. For an AWGN channel, $a_k = 1$.

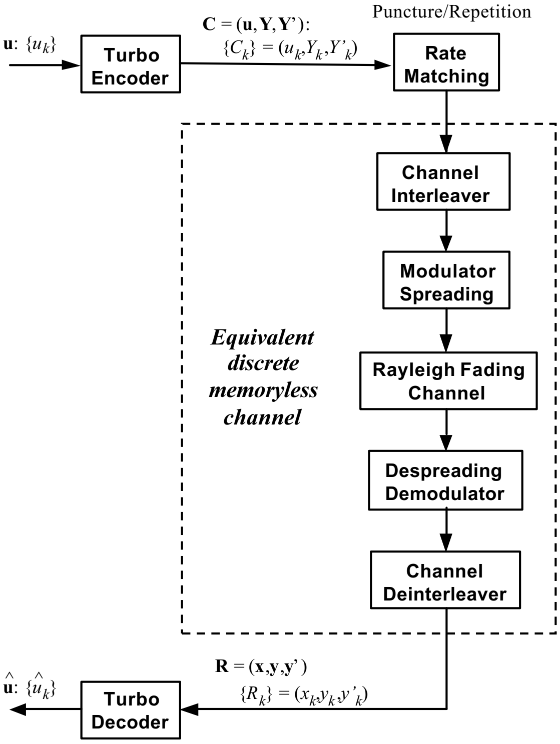

Fig. 1: *HSDPA transmission model.*

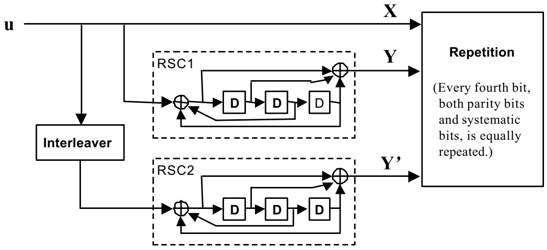

Fig. 2: *Turbo encoder with repetition.*

The global turbo decoder structure, as shown in Fig. 3, includes two constituent decoders, DEC1 and DEC2, implementing *a posteriori* probability, and interleavers/deinterleavers with the same interleaving rule used in the encoder. A soft output algorithm, such as SOVA or Log-MAP, can be applied to the constituent decoders. Let $\Lambda(u_k)$ be the soft output generated by each constituent decoder for the decoded bit u_k. It may be decomposed into the following three terms:

$$\Lambda(u_k) = L_c x_k + L_a(\hat{u}_k) + L_e(\hat{u}_k) \tag{3}$$

where $L_c = 2a_k/\sigma^2$ is the channel reliability value. $L_a(\hat{u}_k)$ is the *a priori* information, which is the extrinsic information provided by the other constituent decoder in the last step of the decoding process. (It is usually set 0 at the beginning of the iterative decoding process). And $L_e(\hat{u}_k)$ is the newly generated extrinsic information by the current constituent decoder. The iterative steps will continue with ever-updating extrinsic information to be exchanged between the two decoders until a reliable hard decision can be made.

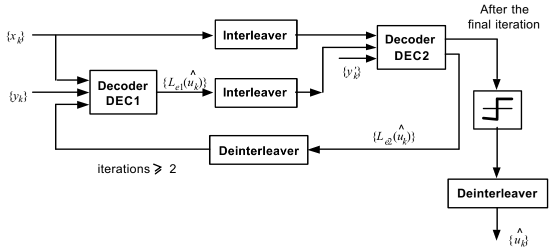

Fig. 3: *Turbo decoder.*

III. The Improved SOVA Using Scaling Approach

Principle of Normalization of Extrinsic Information

For an AWGN channel, it is justified to assume that the soft outputs of a SOVA decoder have Gaussian distribution [1, 7].

For simplicity, for any time step, let v denote the SOVA soft output to be concerned. If an information bit $u = 1$ is transmitted we then have

$$p(v|u=1) = \frac{1}{\sqrt{2\pi}\sigma_v} \cdot e^{-\frac{1}{2\sigma_v^2}(v-m_v)^2}, \tag{4}$$

where m_v and σ_v represent the mean and variance of v, respectively, i.e.,

$$m_v = E\{v\}, \quad \sigma_v = \sqrt{E\{v^2\} - E\{v\}^2}\ . \tag{5}$$

Let

$$L_{\hat{u}} = L_{\hat{u}=1|v}(\hat{u}) = \ln\frac{P(\hat{u}=1|v)}{P(\hat{u}=0|v)} \tag{6}$$

denote the conditional LLR (log-likelihood ratio), given the observation of v, and suppose $P(u = 1) = P(u = 0)$. Using Bayes' rule and eq. (4), we obtain

$$\begin{aligned} L_{\hat{u}} &= \ln\frac{P(v|u=1)}{P(v|u=0)} &(7)\\ &= \ln\left[e^{-\frac{1}{2\sigma_v^2}((v-m_v)^2-(v+m_v)^2)}\right] &(8)\\ &= m_v\frac{2}{\sigma_v^2}v\ . &(9)\end{aligned}$$

This indicates that the soft output v has to be multiplied by the factor

$$s = m_v\frac{2}{\sigma_v^2} \tag{10}$$

to obtain the LLR (unless $s = 1$).

The factor s depends strongly on the BER of the decoder output. Ideally it would be equal to 1. But for a bad channel (low SNR), s is less than 1. In other words, the SOVA is on average too optimistic in its estimate of the reliability.

Simplified Scaling Approach for SOVA Decoding

As mentioned above, a scaling factor, as shown in eq. (scaling-facotr), should be employed to normalize the soft output of SOVA decoder. Usually, the scaling factor depends on SNR. However, in practice, to calculate the mean and variance of the SOVA soft output, multiplication and addition operations must be performed at each symbol-processing cycle during each iteration. Also, to calculate the final scaling factor, a division operation must be done before next iteration begins. All of these imply that a practical SOVA-based turbo decoder with the normalization process embedded may work either with a large clock cycle or with a considerable extra latency when pipeline technique is employed. Therefore, the overall hardware could be probably be comparable with MAP-based turbo decoders [8]. For simple implementation, here we consider the scaling factor as a constant over the entire information sequence and throughout the iterations.

For the rate 1/2 RSC code used in Fig. 2, each transition in the trellis diagram corresponds to one information bit u_k. So for any trellis state at time k, denoted s_k, $s_k \in \{0, 1, \ldots, 2^{\nu}-1\}$, there are two branches entering it with distinct information bits $u_k = 0$ and $u_k = 1$. Let l_i, $i = 1$, 2, be two paths terminating at state s_k with transitions from the previous states $s_{k-1}^{(l_1)}$ and $s_{k-1}^{(l_2)}$. The cumulative metrics associated with the paths can be calculated by

$$\begin{aligned} M^{(l_i)}(s_k) = M(s_{k-1}^{(l_i)}) &+ (2u_k^{(l_i)}-1)L_{c-\mathrm{SOVA}}x_k^{(l_i)} \\ &+ (2u_k^{(l_i)}-1)L_{c-\mathrm{SOVA}}y_k^{(l_i)} \\ &+ (2u_k^{(l_i)}-1)L_{a-\mathrm{SOVA}}(u_k), \qquad i = 1, 2, \end{aligned} \tag{11}$$

and

$$\begin{aligned} M^{(l_i)}(s_k) = M(s_{k-1}^{(l_i)}) &+ (2u_k^{(l_i)}-1)L_{c-\mathrm{SOVA}}x_k^{(l_i)} \\ &+ (2u_k^{(l_i)}-1)L_{c-\mathrm{SOVA}}{y'_k}^{(l_i)} \\ &+ (2u_k^{(l_i)}-1)L_{a-\mathrm{SOVA}}(u_k), \qquad i = 1, 2, \end{aligned} \tag{12}$$

for DEC1 and DEC2, respectively, where $u_k^{(l_i)}$ is the information bit along the path l_i at time k. The metric difference between the survivor path and the concurrent path among above candidate paths is defined by

$$\Delta_k^{(l)} = \frac{1}{2}\left[\max_{i\in\{1,2\}}\{M^{(l_i)}(s_k)\} - \min_{i\in\{1,2\}}\{M^{(l_i)}(s_k)\}\right] \tag{13}$$

When Viterbi decoding is finished, a ML path is selected. Suppose δ is the decoding depth. At time k, a hard decision for u_k can be obtained at time $k+\delta$. Along the ML path, there are $\delta + 1$ nonsurviving paths with those with indices $0, 1, \ldots, \delta$ having been discarded. Denote the differences between their metrics and corresponding surviving paths by $\Delta_{k+1} \geq 0$, $i = 0, 1, \ldots, \delta$. Define a subset D from such metric difference set when two paths have different hard decision. The soft-output of SOVA for the decoded u_k, denoted $\Lambda_{\mathrm{SOVA}}(u_k)$ (in correspondence with the notation in Section II), is represented by

$$\Lambda_{\mathrm{SOVA}}(u_k) = (2\hat{u}_k - 1) \cdot \min_{k+i \in D} \Delta_{k+i} \ . \tag{14}$$

Like (3), $\Lambda_{\mathrm{SOVA}}(u_k)$ can be also represented by

$$\Lambda_{\mathrm{SOVA}}(u_k) = L_{c-\mathrm{SOVA}}x_k + L_{a-\mathrm{SOVA}}(\hat{u}_k) + L_{e-\mathrm{SOVA}}(\hat{u}_k) \ . \tag{15}$$

Therefore, the extrinsic information is extracted from SOVA as

$$L_{e-\mathrm{SOVA}}(\hat{u}_k) = \Lambda_{\mathrm{SOVA}}(u_k) - L_{c-\mathrm{SOVA}}x_k - L_{a-\mathrm{SOVA}}(\hat{u}_k) \ . \tag{16}$$

When scaling technique is applied, the additional scaling factor is incorporated as follows:

$$L_{e-\mathrm{SOVA}}(\hat{u}_k) = [\Lambda_{\mathrm{SOVA}}(u_k) - L_{c-\mathrm{SOVA-Scaling}}x_k - L_{a-\mathrm{SOVA}}(\hat{u}_k)] \cdot s \ . \tag{17}$$

which is to be used as *a priori* information in the metrics of the succeeding constituent decoder.

IV. SIMULATION RESULTS

We investigate the scaling effect in improving the performance of a turbo code under a WCDMA/HSDPA simulation environment. The simulator parameters are shown in Table 1. The turbo code adopted is the rate-1/4 $(1, 15/13, 15/13)_{\mathrm{oct}}$ code incorporated with repetition manner as mentioned in Section 2, with a traditional block interleaver of length 243 bits. In addition to SOVA with scaling, the conventional SOVA (without scaling) and the Log-MAP algorithms are also used for comparisons. The decoding depth for the SOVA like algorithms takes $\delta = 30$. Eight iterations are done in the iterative decoding.

Tab. 1: Simulation parameters

Carrier Frequency	2GHz
Vehicle Speed	3 km/h
CPICH Power	-10 (dB)*
$\mathrm{E_c/I_{or}}$	-1 (dB)**
$\mathrm{I_{or}/I_{oc}}$	-6 (dB)
Channel Estimation	Ideal
Channel Coding Rate	1/4
Modulation	QPSK
No. of Multicodes Simulated	12
SF	16
Propagation Conditions	AWGN, Pedestrian_A
HSDPA Frame Length	2ms

*: 10% of total transmission power.

**: 80% of total transmission power.

The BER and FER are evaluated against E_c/I_{or}, the ratio of transmitted chip energy to total transmission power. Fig. 4 shows the BER performances of several better evaluated scaling factors for SOVA decoding. Around BER $= 2 \times 10^{-3}$, the scaling factor $s = 0.75$ to $s = 0.8$ can provide about 0.4 dB improvement over the conventional SOVA ($s = 0$), and particularly, SOVA with scaling $s = 0.6$ achieves the largest improvement: for BER of 10^{-3}, it achieves more than 0.5 dB again and the difference between the improved SOVA algorithm and the optimum Log-MAP decoding is now only 0.2 dB, while the conventional SOVA suffers about 0.7 dB performance degradation. As for the FER performances as shown in Fig. 5, $s = 0.65$ offers better improvement: more than 0.7 dB gain at the FER of 7×10^{-3} can be obtained compared to SOVA. Its performance also approaches much closer to that of Log-MAP decoding than SOVA.

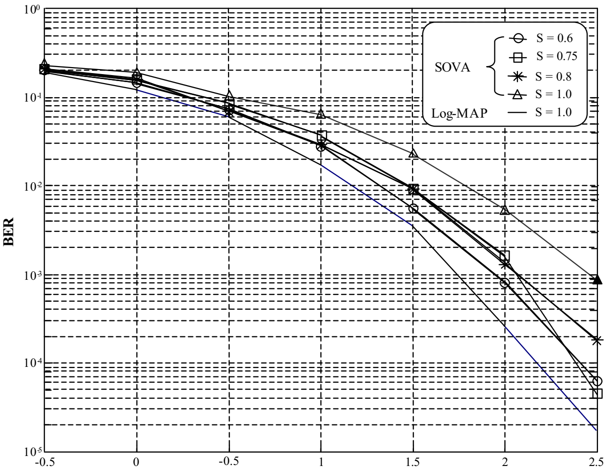

Fig. 4: *BER performance of HSDPA turbo code with different scaling factors.*

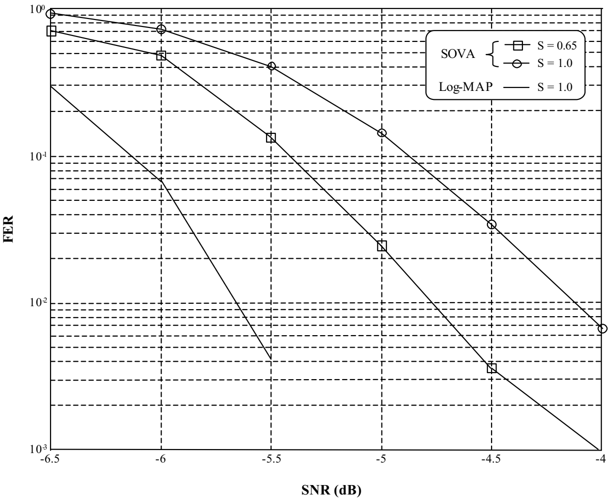

Fig. 5: *FER performance of HSDPA turbo code with different scaling factors.*

V. CONCLUSIONS

In this paper, we have investigated the effect of the scaling approach in improving the performance of SOVA decoding for a turbo code adopted in a WCDMA/HSDPA simulation environment, according to the 3GPP standard. It is observed that incorporating a simple scaling factor during the extrinsic information calculation, coding gains of about 0.5 dB and 0.7 dB can be provided over the conventional SOVA (without scaling) at BER (bit error rate) of $\mathbf{10^{-3}}$ and FER (frame error rate) of $\mathbf{7 \times 10^{-3}}$, respectively. The improved SOVA enjoys the performance much closer to that of the optimum Log-MAP decoding than SOVA.

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