Iterative Channel Estimation for OFDM with Clipping

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Abstract
Estimating a channel that is subject to frequency selective Rayleigh fading is a challenging problem in an orthogonal frequency division multiplexing (OFDM) system with a nonlinear amplifier, e.g., clipping. We propose an iterative algorithm to efficiently estimate the channel impulse response (CIR) of a clipped OFDM system operating in an environment with a multipath fading channel and additive white Gaussian noise (AWGN). A tight lower bound on the mean square error (MSE) of channel estimation is given both by analysis and simulation. The iterative channel estimation algorithm is capable of improving the channel estimate by making use of pilot tones or using the channel estimate of the previous frame to obtain the initial estimate for the iterative procedure. In each iteration, nonlinear distortion is estimated and compensated assuming the nonlinear characteristic is known. Simulation results show that the bit error rate (BER) and MSE of channel estimate can be significantly reduced by this algorithm.

Keywords
OFDM, channel estimation, clipping.

INTRODUCTION
Orthogonal frequency division multiplexing (OFDM) [1], a spectrally efficient form of FDM, divides its allocated channel spectrum into several parallel sub-channels. OFDM is inherently robust against frequency selective fading, since each sub-channel occupies a relatively narrow band, where the channel frequency characteristic is nearly flat. However, it is not possible to make reliable data decisions in OFDM systems unless a good channel estimate is available. Thus, an efficient and accurate channel estimation procedure is necessary to coherently demodulate received data [2]-[5].
Unfortunately, one particular problem with OFDM signals, which is considered a major drawback of OFDM transmission, is its large envelop fluctuations, i.e., large peak-to-average power ratio (PAPR). This Gaussian noise-like amplitude with a very large dynamic range is caused by implementing the inverse fast Fourier transform (IFFT) at the transmitter side. Therefore, it requires RF power amplifiers with a high dynamic range. When transmitted data with high peaks is passed through nonlinear devices or channels, such as clipping in digital-to-analog converters (DAC) or high power amplifier (HPA), the signal may suffer significant spectral spreading, in-band distortion and more critical undesired out-of band radiation. Another serious effect is the increased bit error rate and the difficulty of performing channel estimation in the presence of a nonlinearity because the additional nonlinear distortion creates inter-carrier interference (ICI).
There are two directions to mitigate this large PAPR effect. The obvious and popular one is to reduce PAPR. There are a number of methods to reduce PAPR reported in the literature [6]-[8]. The other direction focuses on the OFDM signal reconstruction with nonlinear effect detection and compensation [9]-[12]. All these signal detection methods assume perfect knowledge of the channel state information. In the case of an unknown channel, the problem becomes one of joint channel estimation and signal detection with nonlinear distortion. We have not found any related paper studying this challenging problem in the literature. In this paper, we propose an iterative channel estimation and signal detection procedure with compensation of the nonlinear effect.

NONLINEAR DISTORTED OFDM SYSTEM MODEL
In this paper, we only consider the nonlinear amplifier at the transmitter side and we only deal with one simple nonlinear distortion-clipping or SL (soft limiter). Nevertheless, the same analysis can be modified for the case where the distortion is caused by the channel or the receiver and more complicated nonlinear amplifiers. The clipped outputs of the time-domain signals are given by

$$x^g(k) = \begin{cases} x(k), & |x(k)| \leq A \\ Ae^{j\arg\{x(k)\}}, & |x(k)| > A \end{cases}, \tag{1}$$

where A is the clipping threshold.
If we suppose the only nonlinear amplifier is at the transmitter, we can write the input-output relation as

$$x^g(k) = x(k) + d^g(k), \tag{2}$$

and obviously $d^g(k) = x^g(k) - x(k)$, where those signals with superscript g represent the corresponding signals with nonlinear distortion triggered by function g [10].
Since the distortion is carried out after the IFFT at the transmitter, the corresponding distorted frequency-domain signals that generate the distorted time-domain signals are given as

$$\begin{aligned} \underline{X}^g &= \mathsf{FFT}\{\underline{x}^g\} \\ &= \mathsf{FFT}\{\underline{x} + \underline{d}^g\} \\ &= \underline{X} + \underline{D}^g. \end{aligned} \tag{3}$$

The corresponding frequency-domain distortion $\underline{D}^g$ represent the in-band distortion components which determine a degradation of the system BER. If the continuous OFDM system is considered, the out-of-band distortion components will affect adjacent frequency bands. In many applications, out-of-band emission might become intolerable even when BER degradation is still acceptable. Reducing out-of-band emission is needed in these situations, e.g., filtering [6]. We only analyze the effect of in-band distortion in this paper.

In the presence of nonlinear amplifier, we may obtain the OFDM system model as

$$Y^g(k) = H(k)\left(X(k) + D^g(k)\right) + N(k), \tag{4}$$

where k is the subcarrier index.

The existence of the in-band distortion $\underline{D}^g$ makes the signal detection difficult when the channel is known because the distortion terms $\underline{D}^g$ are usually unknown or difficult to estimate since ICI $\underline{D}^g$ occurs due to the nonlinear distortion. Moreover, in the case of unknown channel, channel estimation becomes a challenging problem.

A LOWER BOUND OF CHANNEL ESTIMATION FOR CLIPPED OFDM

As stated above, the existence of the nonlinear distortion makes it more difficult to estimate the channel and and it also degrades the channel estimation accuracy. Now we assume the whole frame of OFDM transmitted signals are known at the receiver. Thus, the distortion terms in the frequency domain $\underline{D}^g$ are also known as long as the clipping threshold A is known, since the distortion in the time-domain is deterministic from the time-domain transmitted signals $\underline{x}$. The straightforward method to estimate the channel is least square (LS) estimation

$$\hat{H}^{(1)}(k) = \arg\min_{H(k)} |Y^g(k) - H(k)\left(X(k) + D^g(k)\right)|^2. \tag{5}$$

It is equivalent to obtain $\hat{H}^{(1)}(k)$ by

$$\hat{H}^{(1)}(k) = \frac{Y^g(k)}{X(k) + D^g(k)}. \tag{6}$$

The MSE of the channel estimation using this LS method is

$$\begin{aligned} \mathsf{MSE}(\underline{H}) &= \frac{1}{M}\mathsf{E}\{\|\underline{\hat{H}}^{(1)} - \underline{H}\|_2^2\} \\ &= \mathsf{E}\left\{\left|\frac{Y^g(k)}{X(k) + D^g(k)} - H(k)\right|^2\right\} \\ &= \mathsf{E}\left\{\left|\frac{N(k)}{X(k) + D^g(k)}\right|^2\right\} \\ &= 2\sigma_n^2\mathsf{E}\left\{\frac{1}{|X(k) + D^g(k)|^2}\right\}, \end{aligned} \tag{7}$$

where we omit the subcarrier index k for simple notation.

In order to evaluate (7), the pdf of the random variable $|X(k) + D^g(k)|$ should be derived first. However, it is not a easy task since $\underline{D}^g$ depends on $\underline{X}$. Thus, we first obtain a lower bound on the MSE. We define a new random variable $Z = X(k) + D^g(k)$. Then the MSE of the channel frequency response becomes

$$\mathsf{MSE}(\underline{H}) = 2\sigma_n^2\mathsf{E}\left\{\frac{1}{|Z|^2}\right\}. \tag{8}$$

From the Cauchy-Schwarz inequality, we have the following:

$$1 = \mathsf{E}^2\left\{\frac{1}{Z}Z\right\} \le \mathsf{E}\left\{\frac{1}{Z^2}\right\}\mathsf{E}\{Z^2\}. \tag{9}$$

Thus, we obtain the following lower bound on the MSE

$$\mathsf{MSE}(\underline{H}) \ge 2\sigma_n^2\frac{1}{\mathsf{E}\{|X(k) + D^g(k)|^2\}}. \tag{10}$$

The FFT and IFFT operations preserve the signal energy in the time-domain and the frequency-domain, i.e.,

$$\mathsf{E}\{|X(k) + D^g(k)|^2\} = \mathsf{E}\{|x(k) + d^g(k)|^2\}. \tag{11}$$

Using the reasonable approximation that the time-domain signal amplitudes $|x(k)|$ have a Rayleigh distribution since they are the superposition of M identical independent random variables $\underline{X}$, the above expectation has a closed form

$$\begin{aligned} \mathsf{E}\{|x(k) + d^g(k)|^2\} &= \int_0^A r^2\frac{r}{\sigma_X^2}e^{-\frac{r^2}{2\sigma_X^2}}dr + A^2\mathsf{P}(r > A) \\ &= 2\sigma_X^2\left(1 - e^{-\frac{A^2}{2\sigma_X^2}}\right). \end{aligned} \tag{12}$$

The lower bound of (10) becomes

$$\mathsf{MSE}(\underline{H}) \ge \frac{\sigma_n^2}{\sigma_X^2\left(1 - e^{-\frac{A^2}{2\sigma_X^2}}\right)}. \tag{13}$$

Furthermore, if we know the channel delay spread L, i.e., $\underline{h} = [h_0, \cdots, h_{L-1}, 0, \cdots, 0]^T$, we can further reduce the MSE of the channel frequency response by a factor of $\frac{L}{M}$. Therefore, the final lower bound is give by

$$\mathsf{MSE}(\underline{H}) \ge \frac{L\sigma_n^2}{M\sigma_X^2\left(1 - e^{-\frac{A^2}{2\sigma_X^2}}\right)}. \tag{14}$$

Comparing (10) with the Cramer-Rao lower bound of the channel estimation without nonlinear distortion [17]

$$\mathsf{CRLB}(\underline{H}) = \frac{L\sigma_n^2}{M\sigma_X^2}, \tag{15}$$

we found that the MSE of the channel estimation with nonlinear distortion is approximately degraded by a factor of $1 - e^{-\frac{A^2}{2\sigma_X^2}}$. In fact this degradation comes from the signal power loss due to the clipping.

We constructed an OFDM model to demonstrate the validity and effectiveness of the EM-based signal estimation algorithm. The entire channel bandwidth is 400kHz, and is divided into 64 subcarriers (or tones). To make the tones orthogonal to each other, the symbol duration is 160 μs. An additional 20 μs cyclic prefix is used to provide protection

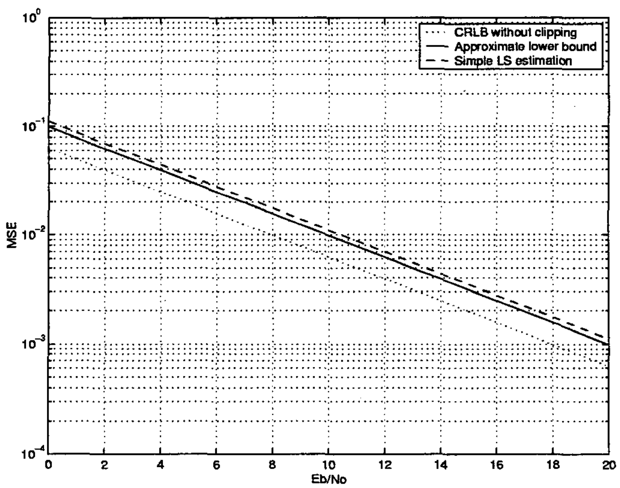

Figure 1. Mean square error v.s. E_b/N_0 when the clipping level is 0dB

from ISI and ICI due to channel delay spread. Thus, the total OFDM frame length is $T_s = 180\ \mu s$ and the subchannel symbol rate is 5.56 kbaud. The modulation scheme used in the system is QPSK. One OFDM frame out of 8 OFDM frames has pilot symbols and 8 pilot symbols are inserted into such a frame. The simulated system can transmit data at 700 kbits/s. The maximum Doppler frequency is chosen to be 55.6Hz, which makes $f_d T_s = 0.01$. The channel impulse response used in the simulation is given by

$$h(n) = \frac{1}{C}\sum_{k=0}^{7} e^{-k/2}\alpha_k \delta(n-k),$$

where $C = \sqrt{\sum_{k=0}^{7} e^{-k}}$ is the normalization constant and $\alpha_k, 0 \le k \le 7$ are independent complex-valued Rayleigh distributed random variables with unit energy, which vary in time according to the Doppler frequency. This is a conventional exponential decay multipath channel model.

Figure 1 shows the MSE performance of the simple LS algorithm. We also show the Cramer-Rao lower bound (CRLB) of non-clipped OFDM system and the approximate lower bound (14). The clipping threshold is $0dB$, i.e., a severe nonlinearity. Several interesting conclusions can be made from this figure. First, the lower bound (14) is a tight bound, which can be used to predict the MSE of OFDM channel estimation with clipping. Second, the MSE degradation due to clipping in OFDM systems is not large, e.g., when the clipping level is $0dB$, the MSE degradation is only about $2dB$. When the clipping level becomes higher, the degradation is even smaller (closer to the CRLB of non-clipped OFDM system). In the next section, we propose an iterative channel estimation algorithm to improve the channel estimation accuracy when only some pilot symbols are known at the receiver.

ITERATIVE CHANNEL ESTIMATION ALGORITHM

Our objective is to estimate the time-domain response $\underline{h}$ or the corresponding frequency-domain response $\underline{H}$ from the observed data $\underline{Y}^g$ with nonlinear distortion. Since OFDM separates the whole frequency selective channel into several parallel frequency flat channels, we only need to estimate the individual $H(k)'s, 0 \le k \le M-1$. To simplify the expressions, we omit the sub-carrier index k in the following expression, and write Y^g, X and H instead of $Y^g(k)$, $X(k)$ and $H(k)$. The initial channel estimate $\underline{H}^{(1)}$ comes from the pilot symbols inserted in the OFDM time-frequency grids. The initial signal estimate can be obtained by

$$X^{(1)} = \text{Hard Decision}\left\{\frac{Y^g}{H^{(1)}}\right\}, \quad (16)$$

$$x^{(1)} = \text{IFFT}(\underline{X}^{(1)}). \quad (17)$$

In the next iteration, we estimate the nonlinear distortion $\underline{D}^{g,1}$ as

$$\begin{aligned}\underline{D}^{g,1} &= \text{FFT}(\underline{d}^{g,1}) \\ &= \text{FFT}(g(\underline{x}^{(1)}) - \underline{x}^{(1)}) \quad (18)\end{aligned}$$

If the $X^{(1)} \approx X$, i.e., the BER is small enough, then $\underline{D}^{g,1} \approx \underline{D}^g$. After we have attained the estimates of the channel and the nonlinear distortion in the p^{th} iteration, the simple LS estimation algorithm and distortion cancellation of the $(p+1)^{st}$ iteration can be carried out and the improved channel estimate and thus signal estimate and nonlinear distortion can be obtain as

$$H^{(p+1)} = \frac{Y^g}{X^{(p)} + D^{g,p}} \quad or \quad (19)$$

$$H^{(p+1)} = \frac{Y^g - H^{(p)}D^{g,p}}{X^{(p)}} \quad (20)$$

$$X^{(p+1)} = \text{Hard Decision}\left\{\frac{Y^g - H^{(p)}D^{g,p}}{H^{(p)}}\right\}, \quad (21)$$

$$x^{(p+1)} = \text{IFFT}(\underline{X}^{(p+1)}), \quad (22)$$

$$\underline{D}^{g,p+1} = \text{FFT}(g(\underline{x}^{(p+1)}) - \underline{x}^{(p+1)}). \quad (23)$$

Figure 2 and 3 show the performance of the proposed iterative channel estimation algorithm when the clipping threshold is $0dB$, which is a very severe clipping case. The effectiveness of the iterative algorithm can be observed from these two figures. However, the BER does not converge to the BER of the known channel nor does the MSE converge to the MSE of the known transmitted signals, which is considered as a lower bound. This is mainly because of the nonlinear amplifier at the transmitter. Furthermore, the performance improvement is gained almost all in the first iteration. The second iteration has only a slight BER improvement and negligible MSE improvement. Further iterations do not show any performance improvement. This leads to the conclusion that one or two iterations is enough to gain the benefit from the nonlinear distortion estimation and cancellation.

Similarly, Figure 4 and 5 show the performance of the proposed iterative channel estimation algorithm when the clip-

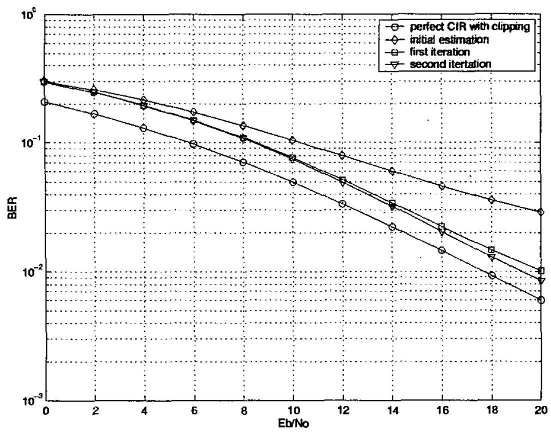

Figure 2. Symbol error rate v.s. E_b/N_0 when the clipping level is 0dB

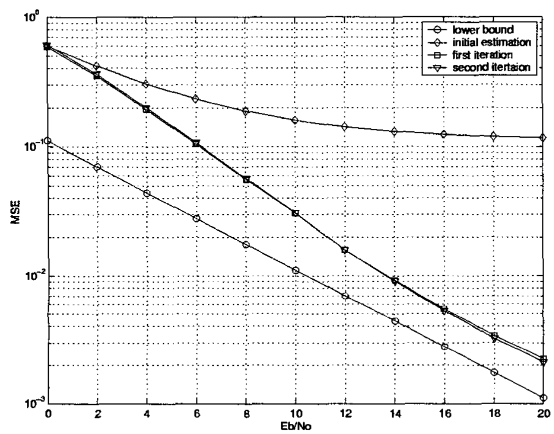

Figure 3. Mean square error v.s. E_b/N_0 when the clipping level is 0dB

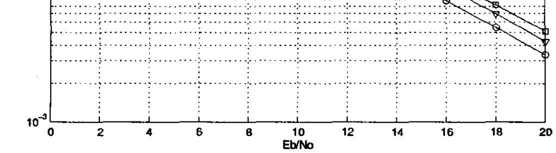

Figure 4. Symbol error rate v.s. E_b/N_0 when the clipping level is 3dB

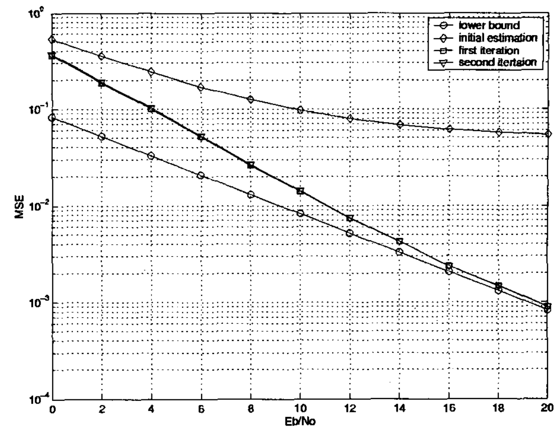

Figure 5. Mean square error v.s. E_b/N_0 when the clipping level is 3dB

ping threshold is $3dB$, which is a moderate clipping case. Similar conclusions can be draw as the above severe clipping case. Furthermore, the larger the clipping level, the closer the performance to the lower bounds in terms of BER and MSE.

CONCLUSION

We have proposed an iterative algorithm to efficiently estimate the channel impulse response (CIR) of a clipped OFDM system operating in an environment with a multipath fading channel and additive white Gaussian noise (AWGN). A lower bound for the MSE of channel estimation is given by analysis and simulation. It is a tight lower bound.The iterative channel estimation algorithm is capable of improving the channel estimate by making use of pilot tones or using the channel estimate of the previous frame to obtain the initial estimate for the iterative procedure. In each iteration, nonlinear distortion is estimated and compensated. Simulation results show that the BER and MSE of channel estimate can be significantly reduced by this algorithm. Most performance improvement is attained in the first iteration, which is another merit of the algorithm. Moreover, The larger the clipping level, the closer the performance to the lower bounds. We will introduce channel coding schemes to further improve the accuracy of channel estimation in our future work.

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REFERENCES

[1] L. J. Cimini, Jr., " Analysis and simulation of a digital mobile chaneel using orthogonal frequency division

multiplexing,"IEEE Transactions on Communications, COM-33, July 1985, pp. 665-675.

[2] Ye (Geoffrey) Li, Leonard J. Cimini, Jr., and Nelson R. Sollenberger, "Robust channel estimation for OFDM systems with rapid dispersive fading channels," IEEE Transactions on Communication, Vol. 46, No. 7, July 1998, pp. 902-915.

[3] Jae Kyoung Moon and Song In Choi, "Performance of channel estimation methods for OFDM systems in multipath fading channels," IEEE Transactions on Consumer Electronics, Vol. 46, No. 1, Feb. 2000, pp. 161-170.

[4] J.-J. van de Beek, O. Edfors, M. Sandell, S. K. Wilson, and P. O. Borjesson, "On channel estimation in OFDM systems," IEEE Vehicular Technology Conference, 1995, Vol. 2, pp. 815 -819.

[5] O. Edfors, M. Sandell, S. K. Wilson , J.-J. van de Beek, and P. O. Borjesson, "OFDM channel estimation by singular value decomposition," IEEE Transactions on Communications, Vol. 46 No. 7, July 1998, pp. 931 -939.

[6] X. Li and L. J. Cimini, Jr., "Effects of clipping and filtering on the performance of OFDM," IEEE Communications Letters, Vol. 2, No. 5, May 1998, pp. 131-133.

[7] L. J. Cimini, Jr., and N. R. Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences," IEEE Communications Letters, Vol. 4, No. 3,Mar. 2000, pp. 86-88.

[8] T. A. Wilkinson and A. E. Jones, "Minimization of the peak to mean envelop power ration of multicarrier transmission schemes by block coding," Vehicular Technology Conference, May 1995, pp. 825-829.

[9] D. Kim and G. L. Stuber, "Clipping noise mitigation for OFDM by decision-aided reconstruction," IEEE Communication Letters, Vol. 3, No. 1, Jan. 1999, pp. 4-6.

[10] J. Tellado, L. M. C. Hoo and J. M. Cioffi, "Maximum likelihood detection of nonlinear distorted multicarrier symbols by iterative decoding," Global Telecommuncations Conference, 1999, Vol. 3, pp 2493-2498.

[11] J. Tellado, "Multicarrier modulation with low PAR – applicaiton to DSL and wireless", Kulwer Academic Publication, 2000

[12] D. Declercq and G.B. Giannakis, "Recovering clipped OFDM Symbols with Bayesian inference," IEEE International Conference on Acoustics, Speech, and Signal Processing, 2000, Vol. 1, pp. 157 -160.

[13] H. Ochiai and H. Imai, "On the distribution of the peak-to-average power ratio in OFDM signals," IEEE Transactions on Communications, Vol. 49, No. 3, Feb. 2001, pp. 282-289.

[14] D. Dardari, V. Tralli, and A. Vaccari, "A theoretical characterization of nonlinear distribution effects in OFDM systems," IEEE Transactions on Communications, Vol. 48, No. 10, Oct. 2000, pp. 1755-1764.

[15] P. Banelli and S. Cacopardi, "Theoretical analysis and performance of OFDM signals in nonlinear AWGN channels." it IEEE Transactions on Communications, Vol. 48, No. 3, Mar. 2000, pp. 430-441.

[16] G. Santella and F. Mazzenga, "A hybrid analytical-simulation procedure for performance evaluation in M-QAM-OFDM schemes in presence of nonlinear distortions", IEEE Transactions on Vechicular Technology, Vol. 47. No. 1, Feb. 1998, pp. 142-151.

[17] X. Ma, H. Kobayashi, and S. Schwartz, "EM-Based Channel Estimation for OFDM," Technical report, Princeton University, June 2002.