

# Maximum Likelihood Channel Estimation and Signal Detection for OFDM Systems

Pei Chen\* and Hisashi Kobayashi

Department of Electrical Engineering

Princeton University

Princeton, New Jersey 08544, USA

**Abstract - We apply joint maximum likelihood (ML) estimation to orthogonal frequency division multiplexing (OFDM) systems and develop a simple receiver structure that gives joint ML estimates of a multipath channel and the transmitted data sequence. Simulation results have confirmed good performance of our algorithm. For a two-path or three-path slow fading channel, our algorithm converges to the case with known channel parameters.**

## 1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [3] is a promising multicarrier digital communication technique for transmitting high bit-rate data over wireless communication channels. OFDM has been chosen for digital audio broadcasting (DAB) and digital video broadcasting (DVB). It is also the technique used for the standards for wireless 5GHz local area networks (IEEE 802.11a in the U.S. and HIPER-LAN/2 in Europe).

Channel estimation for OFDM systems has been an active research subject [1] because of its substantial influence on the overall system performance. Insertion of pilots in OFDM symbols provides a base for reliable channel estimation. For slow fading channels, channel attenuations for different subchannels are correlated within one OFDM symbol and across several OFDM symbols. When pilots are transmitted in certain positions in the time and frequency grid of OFDM, channel estimation can be performed by two-dimensional interpolation. One type of such estimation algorithms uses fixed parameter linear interpolation [10,12]. These algorithms are very simple to implement, but a large estimation error is inevitable in case of model mismatch. If the statistical properties of the channel are known, an optimal linear channel estimator in the minimum mean-squared error (MMSE) sense can be designed by using a two-dimensional Wiener filter [6]. To reduce the computational complexity involved in two-dimensional filters, suboptimal low-complexity separable estimators, which use one-dimensional finite impulse response Wiener filters in the time and frequency directions separately

have been suggested [6]. A robust implementation of the MMSE pilot symbol assisted estimator, which does not depend on channel statistics, has been discussed in [9].

Decision directed estimation is a type of blind approach. Considering individual OFDM symbols, a low complexity approximation to the frequency-based linear minimum mean-squared error (LMMSE) estimator has been proposed in [4]. This algorithm uses singular value decomposition and the theory of optimal rank reduction. A related algorithm, which takes advantage of the channel correlation in the time direction as well, is presented in [8]. A second type of low complexity approximation to the LMMSE estimator regarding individual OFDM symbols is based on using transforms that concentrate the channel power to a few coefficients in the time domain. Low-complexity estimators of this type, based on both the DFT and optimal rank reduction, have been proposed in [13].

Several other blind channel estimation algorithms have also been devised for OFDM systems. Some of them are based on a subspace approach exploiting the cyclostationary property that is inherent to OFDM transmissions in the cyclic prefix [5,11]. Another type of blind channel estimator capitalizes on the finite alphabet property of the modulated symbols [14].

In this paper, we apply the method of maximum likelihood (ML) estimation to OFDM systems, and develop ML estimation algorithms to estimate jointly the multipath fading channel and the transmitted data sequence. The algorithm works in an iterative fashion: it computes an initial estimate of the channel based on either the pilot symbols or the estimate obtained from the previous OFDM symbol and then operates in a decision directed mode. The algorithm does not require any prior knowledge about the channel. Exploiting the properties of OFDM systems gives a very simple structure to realize the joint ML estimate.

In what follows, we will first describe the OFDM system model, including the effects of a multipath channel. Then we derive an iterative algorithm that performs joint ML estimate of the channel and the data sequence, and present simulation results that verify the performance of the algorithm. Part of

\*Currently with Qualcomm Inc.

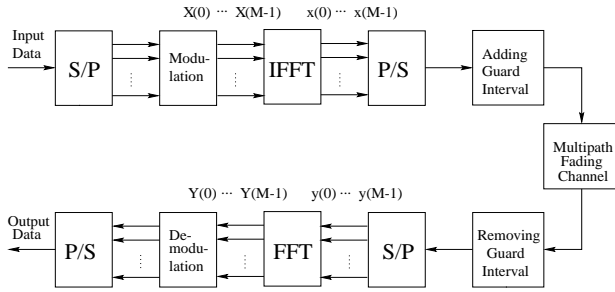


Figure 1: Baseband OFDM System

the work has been published in [2].

## 2. OFDM SYSTEM MODEL

Figure 1 shows a baseband equivalent representation of a typical OFDM system. We focus our discussion on estimation of one OFDM symbol instead of a sequence of symbols for the reason to be justified below. At the transmitter side, the serial input data is converted into  $M$  parallel data streams, and each data stream is modulated by a linear modulation scheme, such as QPSK, 16QAM or 64QAM. If QPSK is used, for instance, the binary input data of  $2M$  bits will be converted into  $M$  QPSK symbols by the serial-to-parallel converter (S/P) and the modulator. The modulated data symbols, which we denote by complex-valued variables  $X(0), \dots, X(m), \dots, X(M-1)$ , are then transformed by the IFFT, and the complex-valued outputs  $x(0), \dots, x(k), \dots, x(M-1)$  are converted back to serial data for transmission. A guard interval is inserted between the symbols. If the guard interval is longer than the channel delay spread, and if we discard the samples of the guard interval at the receiving end, the ISI will not affect the actual OFDM symbol. Therefore, the system can be analyzed on symbol-by-symbol basis. At the receiver side, after converting the serial data to  $M$  parallel streams, the received samples  $y(0), \dots, y(k), \dots, y(M-1)$  are transformed by the FFT into  $Y(0), \dots, Y(m), \dots, Y(M-1)$ , which should be equivalent to the data symbols  $X(0), \dots, X(m), \dots, X(M-1)$  in the absence of channel distortion and/or noise. They are then demodulated and re-ordered in a serial order.

We consider a multipath channel model with the length of its impulse response at most  $L$  time units, where the time unit is  $\frac{2M}{R(M+N)}$  for QPSK modulation. Here  $R$  is the source data rate;  $M$  is the number of subcarriers; and  $N$  is the length of the guard interval. Using the notation for OFDM symbols, the output of the channel can be written as

$$y(k) = \sum_{l=0}^{L-1} h_l x(k-l) + n(k), \quad 0 \leq k \leq M-1. \quad (1)$$

where  $(h_0, \dots, h_{L-1})$  is the channel impulse response; and  $n(k)$  is the additive white Gaussian noise. Note that  $y(k)$ ,  $x(k)$ ,  $n(k)$  and  $h_l$  are all complex valued.

This model essentially assumes that the channel is slowly fading, i.e. the channel is constant during one OFDM symbol. As will be discussed in Section 4.1, a time-varying channel can be well approximated by a constant channel during a time interval  $T$  if  $f_d T \leq 0.01$  is satisfied, where  $f_d$  is the maximum Doppler frequency, and in our case  $T$  is taken as the OFDM symbol interval. If the source data rate is  $R$  bits/second, and the modulation scheme is QPSK, we have  $T = \frac{2M}{R}$ . For a vehicle moving at speed  $v$ , the maximum Doppler frequency is  $f_d = f_c \frac{v}{c}$  where  $f_c$  is the carrier frequency and  $c$  is the speed of light. Hence, the relationship that we need to satisfy the assumption is

$$M \leq 0.01 \times \frac{R}{2} \times \frac{c}{v} \times \frac{1}{f_c}. \quad (2)$$

For example, if the data rate is 2Mbps, the speed of the vehicle is 66mph and the carrier frequency is 1GHz, then we can safely assume the channel characteristic remains unchanged during one OFDM symbol, insofar as  $M$  is less than 100.

If cyclic prefix is used for the guard interval, intercarrier interference in a multipath channel can be also avoided. Then it can be shown that the following simple relation between  $Y(m)$  and  $X(m)$  holds:

$$\begin{aligned} Y(m) &= \left( \sum_{l=0}^{L-1} h_l \exp^{-j2\pi \frac{ml}{M}} \right) X(m) + N(m) \quad (3) \\ &= H(m)X(m) + N(m), \quad 0 \leq m \leq M-1, \quad (4) \end{aligned}$$

where  $H(m)$  is the complex frequency response of the channel at subchannel  $m$ , and  $N(0), \dots, N(M-1)$  are the DFT of  $n(0), \dots, n(M-1)$ . If  $n(0), \dots, n(M-1)$  are i.i.d. Gaussian random variables, so are the transformed variables  $N(0), \dots, N(M-1)$ . Equation (4) shows that the received signal is the transmitted signal attenuated and phase shifted by the frequency response of the channel at the subchannel frequencies and disturbed by noise.

## 3. ITERATIVE ML ESTIMATION ALGORITHM

We can solve the channel estimation and signal detection problem by using Eq. (3) or (4). The channel frequency response parameters  $H(0), \dots, H(M-1)$  are generally correlated among each other, whereas the impulse response parameters  $h_0, \dots, h_{L-1}$  may be independently specified, thus the number of parameters in the time domain is smaller than that in the frequency domain. Therefore, it is more appropriate to apply the ML algorithm to (3), i.e., find ML estimate of the channel in the time domain.

We consider joint estimation of the channel and the transmitted signal. To simplify notation, we use  $\underline{X}$ ,  $\underline{h}$  and  $\underline{Y}$

to represent the transmitted signal, the channel impulse response and the received signal, respectively. The likelihood function of  $\underline{Y}$ , given  $\underline{X}$  and  $\underline{h}$ , is

$$f(\underline{Y}|\underline{X}, \underline{h}) = \frac{1}{(2\pi\sigma^2)^M} \exp\left\{-\frac{D(\underline{h}, \underline{X})}{2\sigma^2}\right\}, \quad (5)$$

where  $\sigma^2$  is the variance of both real and imaginary components of  $n(k)$  and is equivalent to  $\frac{1}{2}E[|n(k)|^2]$ , and the function  $D(\underline{h}, \underline{X})$ , which we call the ‘‘distance’’ function, is defined as

$$D(\underline{h}, \underline{X}) = \sum_{m=0}^{M-1} |Y(m) - \sum_{l=0}^{L-1} h_l \exp^{-j2\pi \frac{ml}{M}} X(m)|^2. \quad (6)$$

We need to find  $\underline{h}$  and  $\underline{X}$  that jointly maximize  $f(\underline{Y}|\underline{X}, \underline{h})$ , or equivalently, minimize the distance function  $D(\underline{h}, \underline{X})$ .

Let  $h_l = a_l + jb_l$  for  $0 \leq l \leq L-1$ . If we know  $\underline{X}$ , we can solve for  $h_l$  by

$$\frac{\partial D(\underline{h}, \underline{X})}{\partial a_l} \Big|_{\underline{h}=\hat{\underline{h}}} = 0, \quad (7)$$

$$\frac{\partial D(\underline{h}, \underline{X})}{\partial b_l} \Big|_{\underline{h}=\hat{\underline{h}}} = 0, \quad (8)$$

which readily lead to

$$\sum_{l=0}^{L-1} \hat{a}_l \Re\{s(k-l)\} - \sum_{l=0}^{L-1} \hat{b}_l \Im\{s(k-l)\} = \Re\{z(k)\}, \quad (9)$$

$$\sum_{l=0}^{L-1} \hat{a}_l \Im\{s(k-l)\} + \sum_{l=0}^{L-1} \hat{b}_l \Re\{s(k-l)\} = \Im\{z(k)\}, \quad (10)$$

for  $0 \leq k \leq L-1$ , or equivalently,

$$\sum_{l=0}^{L-1} \hat{h}_l s(k-l) = z(k), \quad 0 \leq k \leq L-1, \quad (11)$$

where  $z(k)$  and  $s(k)$  are defined as the IDFT of

$$Z(m) = X^*(m)Y(m), \quad 0 \leq m \leq M-1, \quad (12)$$

and

$$S(m) = |X(m)|^2, \quad 0 \leq m \leq M-1, \quad (13)$$

respectively.

If we take the DFT of size  $L$  on both sides of Eq. (11), we have,

$$\hat{H}^{(L)}(l)S^{(L)}(l) = Z^{(L)}(l), \quad 0 \leq l \leq L-1, \quad (14)$$

where the superscript  $(L)$  denotes the size of DFT to distinguish from the previous DFT and IDFT, which are all of size  $M$ . Thus  $\hat{h}_l$  can be obtained as the size  $L$  IDFT of  $Z^{(L)}(l)/S^{(L)}(l)$  for  $0 \leq l \leq L-1$ , i.e.,

$$\hat{\underline{h}} = \text{IDFT} \left\{ \frac{Z^{(L)}}{S^{(L)}} \right\}. \quad (15)$$

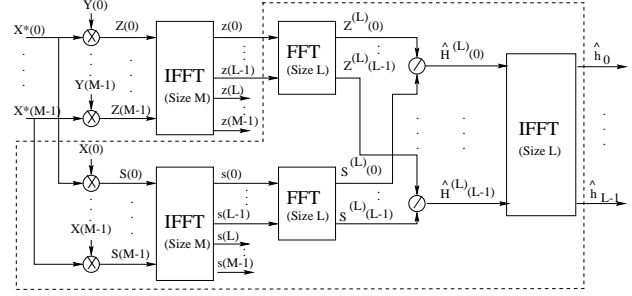


Figure 2: Channel Estimation for OFDM

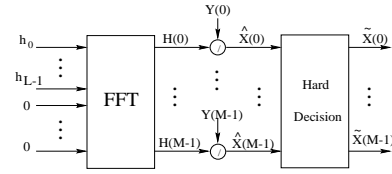


Figure 3: Signal Detection

For constant modulus signals, we have  $|X(m)|^2 = C$  for all  $m$ , where  $C$  is a constant. Therefore,

$$s(k) = \begin{cases} C, & k = 0 \\ 0, & k \neq 0 \end{cases} \quad (16)$$

In this case, from Eq. (11) we can directly obtain

$$\hat{h}_k = z(k)/C, \quad 0 \leq k \leq L-1. \quad (17)$$

Hence, for given  $\underline{X}$ , the ML estimate of the channel  $\hat{\underline{h}}$  is the solution given by Eq. (15) or (17).

One problem with the above algorithm is the unknown channel memory length  $L$ . However, since the system requires that the channel memory be no greater than the guard interval  $N$ , we can satisfy this requirement by setting  $L = N$ .

Figure 2 shows the diagram of the above ML channel estimation procedure. The steps shown in blocks with dashed lines can be replaced by a division of a constant  $C$  for constant modulus signals.

For a given channel impulse response  $\underline{h}$  or its frequency response  $\underline{H}$ , the ML estimate of the transmitted signal can be obtained by

$$\begin{aligned} \hat{X}(m) &= \operatorname{argmin}_{\underline{X}=\hat{\underline{X}}} \{D(\underline{h}, \underline{X})\} \\ &= Y(m)/H(m), \quad 0 \leq m \leq M-1. \end{aligned} \quad (18)$$

$\hat{X}(0), \dots, \hat{X}(M-1)$  is then passed through a hard decision block, which generates the detected signal  $\tilde{X}(0), \dots, \tilde{X}(M-1)$ . The signal detection diagram is shown in Fig. 3.

The channel estimation and signal detection procedures described above are used iteratively to find the joint ML estimates. The joint estimation starts from OFDM symbols that

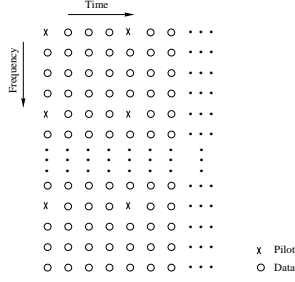


Figure 4: An example of transmitted OFDM symbols

contain some pilot symbols. An initial ML estimate of the impulse response of the channel is obtained solely by the pilot symbols. Based on the initial estimate of the channel, the first estimation of the transmitted signal can be calculated. Both the pilot symbols and the estimated transmitted signals are then fed back to the channel estimation step to obtain an improved estimation of the channel. Then an updated estimation of the transmitted signal can be obtained using the new channel estimate. The iteration procedure stops when the incremental improvement on the channel estimation is below a pre-determined threshold. For those OFDM symbols which do not contain pilot symbols, the iteration starts by setting the final estimation in the previous OFDM symbol as the initial estimation of the channel. The rest of the iteration follows the same procedure as done for OFDM symbols with pilot.

More precisely, using the OFDM symbol structure shown in Fig. 4 as an example (where each column represents an OFDM symbol), the iterative algorithm is described as follows.

A. Initialization step: Set  $i = 0$

A-1. Initial estimate for symbols with pilots:

Use the pilot symbols to find  $\hat{\underline{h}}^{(1)}$  that minimizes the distance function  $D(\underline{h}, \underline{X}^{(0)})$ , i.e.,

$$\hat{\underline{h}}^{(1)} = \underset{\underline{h}}{\operatorname{argmin}} \{D(\underline{h}, \underline{X}^{(0)})\}, \quad (19)$$

where  $D(\underline{h}, \underline{X}^{(0)})$  is defined by

$$D(\underline{h}, \underline{X}^{(0)}) = \sum_{n=0}^{M/4-1} |Y(4n) - \sum_{l=0}^{L-1} h_l \exp^{-j2\pi \frac{nl}{M/4}} X(4n)|^2. \quad (20)$$

In this step, only a part of the received signals, i.e.,  $Y(0), Y(4), \dots, Y(M-4)$  are utilized. The IFFTs in Fig. 2 are of size  $\frac{M}{4}$ .

A-2. Initial estimate for symbols without pilots:

Set  $\hat{\underline{h}}^{(1)}$  to be the final estimate of the impulse response of the channel obtained from the previous OFDM symbol.

B. Updating step: For  $i \geq 1$

B-1. Signal detection: Given the channel estimate  $\hat{\underline{h}}^{(i)}$ , find  $\hat{\underline{X}}^{(i)}$  that minimizes  $D(\hat{\underline{h}}^{(i)}, \underline{X})$  of Eq. (6), i.e.,

$$\hat{\underline{X}}^{(i)} = \underset{\underline{X}}{\operatorname{argmin}} \{D(\hat{\underline{h}}^{(i)}, \underline{X})\}, \quad (21)$$

and produce a hard decision  $\tilde{\underline{X}}^{(i)}$  by comparing  $\hat{\underline{X}}^{(i)}$  with appropriately set thresholds.

B-2. Channel estimation: Given the decision output  $\tilde{\underline{X}}^{(i)}$ , replace those positions corresponding to pilot symbols with true values and find  $\hat{\underline{h}}^{(i+1)}$  that minimizes  $D(\underline{h}, \tilde{\underline{X}}^{(i)})$  of (6), i.e.,

$$\hat{\underline{h}}^{(i+1)} = \underset{\underline{h}}{\operatorname{argmin}} \{D(\underline{h}, \tilde{\underline{X}}^{(i)})\}. \quad (22)$$

B-3. If the difference between two successive estimates  $|\hat{\underline{h}}^{(i+1)} - \hat{\underline{h}}^{(i)}|$  is below a predetermined threshold, terminate the iteration and output the final decision  $\tilde{\underline{X}}^{(i)}$ ; otherwise, set  $i + 1 \rightarrow i$  and go to step B-1.

## 4. SIMULATION RESULTS

### 4.1 Verification of Constant Channel Assumption

The time variation of the channel is characterized by the normalized Doppler frequency  $f_d T$ . Conventionally, if  $f_d T$  is less than 0.01, the channel can be assumed as constant during the time interval  $T$ . We have verified this assumption using a Rayleigh fading channel model generated by Jakes' model [7].

To quantitatively compare the variation of fading channels with different  $f_d T$  during time interval  $T$ , we have calculated the variance of the magnitude of the fading process with normalized power during the interval  $T$ . The results for different  $f_d T$  is shown in Fig. 5. When  $f_d T = 0.01$ , the variance of the magnitude in time interval  $T$  is about 0.01 for a Rayleigh fading process with normalized power. Therefore, the channel can be considered approximately constant in  $T$ . The figure also shows that the variance grows with  $f_d T$  at a speed proportional to  $(f_d T)^2$ .

### 4.2 System Performance

We have validated our ML channel estimation algorithm by simulation. We use  $M = 64$  subcarriers and QPSK modulation for each subcarrier. We transmit one OFDM symbol with pilots in every four OFDM symbols. For those OFDM symbols containing pilots, we use four equally spaced subcarriers for pilot symbols. Therefore, the pilot-to-data ratio is 1/64. A two-path Rayleigh fading channel with transfer function

$$h(z) = 0.8\alpha_0 e^{j\theta_0} + 0.6\alpha_1 e^{j\theta_1} z^{-1} \quad (23)$$

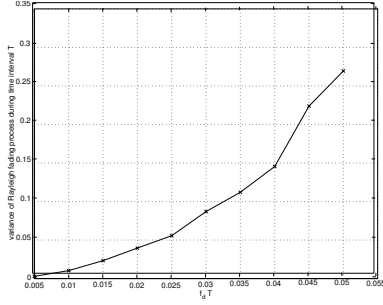


Figure 5: Variance of Rayleigh fading process during interval  $T$  with different normalized Doppler frequency

and a three-path Rayleigh fading channel

$$h(z) = 0.408\alpha_0 e^{j\theta_0} + 0.816\alpha_1 e^{j\theta_1} z^{-1} + 0.408\alpha_2 e^{j\theta_2} z^{-2}, \quad (24)$$

have been used for simulations. Here  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$  are i.i.d. random variables with Rayleigh distribution, and  $\theta_0$ ,  $\theta_1$  and  $\theta_2$  are i.i.d. random variables with uniform distribution. The BER performance for different SNRs when the normalized fading parameter  $f_d T = 0.01$  is shown in Fig. 6. The corresponding mean squared error (MSE) of the estimated channel parameters are shown in Fig. 7. The BER performance is

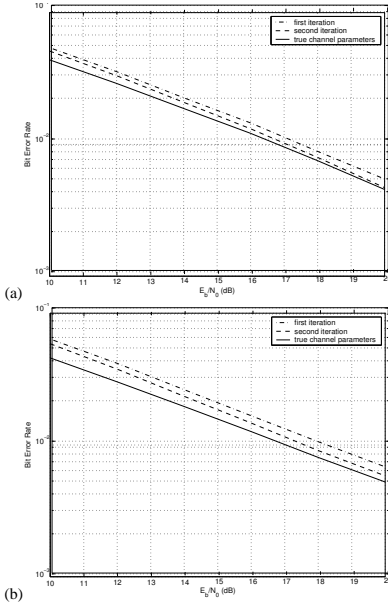


Figure 6: BER v.s.  $E_b/N_0$  for Rayleigh fading channels with  $f_d T = 0.01$  (a) two-path channel (b) three-path channel

compared with ideal cases, where the channel parameters are exactly known at the receiver. Our simulation results show that for SNRs between 10dB and 20dB the second iteration can result in a 0.3dB to 0.8dB gain over the first iteration, and in the region of high SNR the BER performance of our algorithm is close to that of the ideal cases.

Figures 8 and 9 present the results when time variation

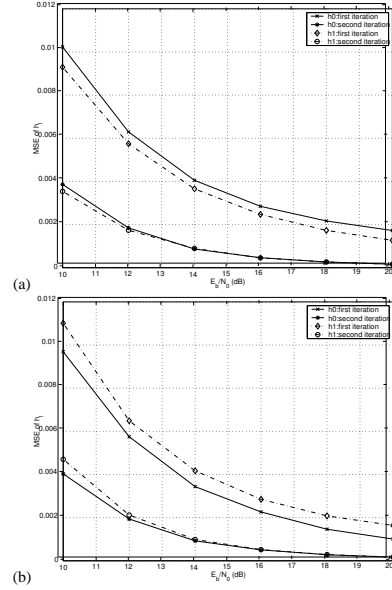


Figure 7: MSE of  $h_l$  v.s.  $E_b/N_0$  for Rayleigh fading channels with  $f_d T = 0.01$  (a) two-path channel (b) three-path channel

of the channel gets larger, where the normalized Doppler frequency  $f_d T$  is 0.05. The mean squared estimation error of the channel parameters in the first iteration is considerably higher compared with the case when  $f_d T = 0.01$ . Moreover, increasing SNR does not decrease the MSE as fast as in the slow fading case, because when  $f_d T$  gets larger, for those OFDM symbols without pilots, the MSE for the initial estimate will be large as the channel changes considerably from one OFDM symbol to another. In the case when channel changes faster, a larger gain in terms of BER and MSE will result from the second iteration. For example, at  $10^{-2}$  BER, the second iteration will bring the required SNR more than 2dB down. However, the third iteration does not give much improvement. For SNRs between 10dB and 20dB, compared with the ideal cases, the BER performance is about 1dB worse for the two-path channel and about 1-1.5dB worse for the three-path channel.

## 5. CONCLUDING REMARKS

We have proposed an iterative ML estimation algorithm for OFDM systems that jointly estimates the multipath channel and the transmitted data sequence. In each iteration, the ML channel estimate for given data sequence is obtained by processing the pilot symbols and the current estimate of the data sequence with FFTs/IFFTs, whereas the ML estimate of the transmitted data can be obtained independently for each subcarrier using the estimated channel frequency response at the corresponding subcarrier. We have conducted simulation experiments, which showed that, for a two-path or three-path slow fading channel, our algorithm converges to the case with known channel parameters.

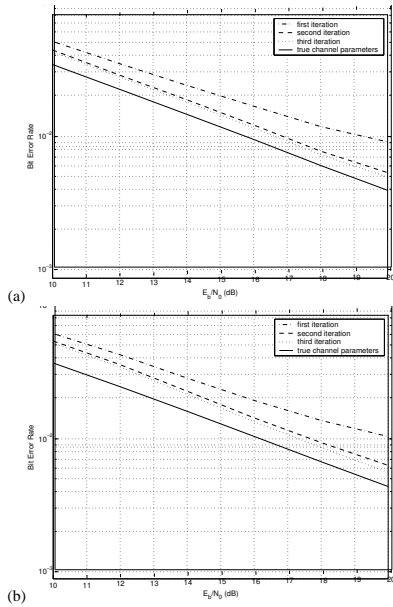


Figure 8: BER v.s.  $E_b/N_0$  for Rayleigh fading channels with  $f_d T = 0.05$  (a) two-path channel (b) three-path channel

#### ACKNOWLEDGMENTS

The present work was supported in part by grants awarded to Princeton University by the New Jersey Center for Wireless Telecommunications (NJCWT), and by Mitsubishi Electric Research Laboratories (MERL), NJ.

#### REFERENCES

- [1] P. Chen, *Signal Detection and Channel Estimation in Multipath Channels*, Ph. D. Thesis, Princeton University, July 2001.
- [2] P. Chen, H. Horng, J. Bao, and H. Kobayashi, "A joint channel estimation and signal detection algorithm for OFDM systems," *Proc. of 2001 Int. Sym. on Signals, Systems, and Electronics (ISSSE'01)*, July 2001.
- [3] L. J. Cimini, Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. COM-33, no. 7, pp. 665-675, July 1985.
- [4] O. Edfors, M. Sandell, J.-J. van de Beek, S.K. Wilson, and P.O. Borjesson, "OFDM channel estimation by singular value decomposition," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931-939, July 1998.
- [5] R. W. Heath Jr. and G. B. Giannakis, "Exploiting input cyclostationarity for blind channel identification in OFDM systems," *IEEE Trans. Sig. Proc.*, vol. 47, no. 3, pp.848-856, Mar. 1999.
- [6] P. Hoeher, S. Kaiser, and P. Robertson, "Two-dimensional pilot-symbol-aided channel estimation by Wiener filtering," *Proc. of 1997 IEEE Int. Conf. on Acoustics, speech, and Signal Processing (ICASSP-97)*, vol. 3, pp. 1845-1848, Apr. 1997.

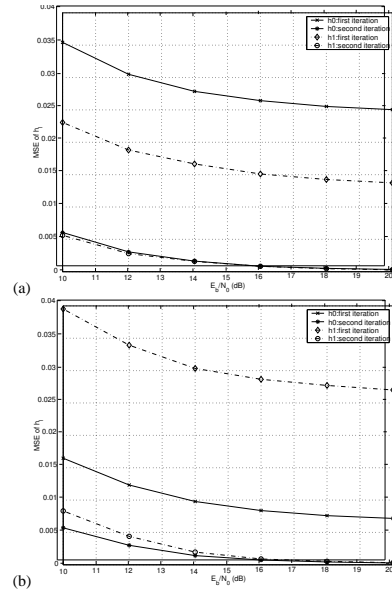


Figure 9: MSE of  $h_1$  v.s.  $E_b/N_0$  for Rayleigh fading channels with  $f_d T = 0.05$  (a) two-path channel (b) three-path channel

- [7] W. C. Jakes, "Multipath Interference", Chapter 1 of *Microwave Mobile Communications*, W. C. Jakes, Ed., IEEE Press, 1994.
- [8] Y. Li, L. J. Cimini, Jr., and N. R. Sollenberger, "Robust channel estimation for OFDM systems with rapid dispersive fading channels," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 902-915, July 1998.
- [9] Y. Li "Pilot-symbol-aided channel estimation for OFDM in wireless systems," *IEEE Trans. Veh. Technol.*, vol. 49, no. 4, pp. 1207-1215, July 2000.
- [10] J.K. Moon and S.I. Choi, "Performance of channel estimation methods for OFDM systems in a multipath fading channels," *IEEE Trans. Consumer Electronics*, vol. 46, no. 1, pp. 161-170, Feb. 2000.
- [11] B. Muquet, M. de Courville, P. Duhamel, and V. Buzenac, "A subspace based blind and semi-blind channel identification method for OFDM systems," *Prod. IEEE Workshop Sig. Proc. Advances in Wireless Commun. (SPAWC'99)*, pp. 170-173, May 1999.
- [12] F. Said and A. H. Aghvami, "Linear two dimensional pilot assisted channel estimation for OFDM systems," *6th IEE Conf. on Telecommunications*, pp. 32-36, Mar. 1998.
- [13] J.-J. van de Beek, O. Edfors, M. Sandell, S. K. Wilson and P. O. Borjesson, "On channel estimation in OFDM systems," *Proc. of IEEE Vehic. Technol. Conf. (VTC'95)*, vol. 2, pp. 815-819, Sept. 1995.
- [14] S. Zhou and G. B. Giannakis, "Finite alphabet based channel estimation for OFDM and related multicarrier systems," *Proc. of 34th Annual Conference on Information Sciences and Systems (CISS'00)*, pp. WP3-31-36, Mar. 2000.