Error Control of Generalized Concatenated Systems

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Abstract

We study error correcting codes, which are constructed by using "permutation" and "iterative decoding" in conjunction with "concatenation". These codes include "concatenated codes," "product codes" and recently invented "Turbo codes" as special cases.

We then extend our approach to a continuous channel, by adopting AZD (ambiguity zone detection) at the receiver, which introduces generalized erasures. Iterative decoding can significantly improve the performance of a system that contains a channel with memory (e.g., spectral shaping, ISI, multipath delay) and/or modulation with memory (e.g., trellis coded modulation, continuous phase modulation), is implementable with low decoding complexity, and requires a small decoding delay.

1 Introduction

"Concatenation" as a technique was first proposed by Forney [4] to connect inner and outer code at the encoder. In many cases an overall optimal decoder may be too complex, and an obvious technique based on one-path decoding in tandem may be far from being optimal.

We explore an "iterative decoder", which can be shown to provide an approximation of an overall optimal decoder, with low complexity in its implementation. We can further improve the overall structure by inserting a "permutation" between the concatenated encoders. Permutation, a more general concept than interleaving, is beneficial not only for channels with burst noise, but also for channels with random noise, and can be introduced without any degradation in information rates.

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2 Generalized Concatenated Systems

In the present paper we extend the classical concept of concatenated codes to a larger class of systems. We define a *"generalized concatenated system"* as a system that can be built by concatenation (serial, parallel or combination) of blocks that include some redundancy or memory. Such blocks include not only channel encoders, as in the conventional concatenated codes, but also

1. a channel with spectral shaping (e.g., partial-response coding) or line coding,
2. a channel with ISI (intersymbol interference) and/or multipath delay,
3. modulation with memory (e.g., trellis coded modulation, continuous phase modulation [1])

We insert permutations between these building blocks. Hence concatenated codes, product codes and recently introduced Turbo codes [3] are treated as special case of the generalized concatenated system.

3 New Decoding Technique

We introduce a novel receiver structure [2] which combines AZD (ambiguity zone detection) and iterative decoding. The idea of AZD (or sometimes called the null zone detector) was successfully applied to a partial response system [6]. A continuous channel with AZD at the receiver front can be viewed as a discrete channel with generalized erasures. In our iterative decoding, AZD provides flexibility of deferring decision on unreliable digits by labeling them as "ambiguous digits" or "generalized erasures".

The iterative decoding technique we introduce is, in concept, similar to iterative decoding procedures used in decoding Turbo codes [3], soft decision decoding scheme discussed by Hagenauer et al. [8] in decoding a concatenated code with Reed-Solomon code and convolutional code. However, the combination of iterative decoding and AZD in the context of the generalized concatenated system is novel to our best knowledge. The iterative decoder makes step-by-step resolutions of these ambiguous digits by capitalizing on the redundancy introduced by the error correcting code, and modulation/channel with memory. The permutation alleviates the clustering effect of residual erasures introduced in the iterative decoding steps.

We present simulation results, which demonstrate a significant performance improvement over previously known approaches such as PRML (partial-response coding, maximum-likelihood decoding). Our iterative decoding technique can be applied to a number of practical systems including digital magnetic recording and wireless communications [2].

4 Example

For an illustration of our scheme, we consider an existing wireless packet transmission system with the following parameters:

- A packet contains 160 information bits.
- The transmitter (Figure 1) first encodes the data by an error correcting code (ECC) - a shortened Hamming (12, 8) code. Resulting 240 bits are permuted by a 20 × 12 block interleaver and then passed to the duobinary encoder which includes a precoder and a duobinary modulator (G(D)= 1+D).
- To achieve space and time diversity, each packet is transmitted from three different locations, three different times from each location, i.e., there are nine transmission attempts overall.
- The channel is subject to additive white Gaussian noise (AWGN).
- The desired quality of service at the receiver is 99.9 % packet completition rate. This corresponds to about 54 % packet throughput in each transmission attempt.

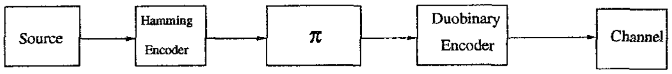

Figure 1: Transmitter from the Example.

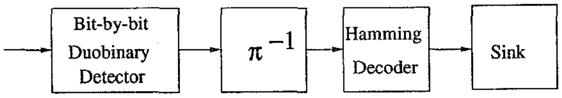

Figure 2: Bit-by-bit detection based receiver.

We consider two conventionally used receivers (Figures 2 and 3) and proposed new iterative receiver (Figure 4). Nine independent trials for successful decoding of each packet are made by each receiver. The first receiver uses a hard-decision bit-by-bit detector for duobinary encoded data followed by a syndrome decoder. The second receiver is a PRML receiver with soft inputs and binary outputs, followed by a syndrome decoder for the Hamming code.

The proposed receiver functions as follows. First, the channel output symbols are passed through an ambiguity zone detector, i.e., a threshold detector that quantizes the data to five levels at the receiver front, as discussed in [6]. Thus in addition to the duobinary symbols 0, 1 and 2, the data passed to the inner decoder contains erasures. The concatenated decoders for the duobinary code and Hamming code form a loop. They are separated by the permutation (in the feedback path) and its inverse-permutation (in the forward path) to preserve the order of the data. These two decoders are capable of performing decoding with erasures. The decoders can be based on soft decision maximum likelihood decoders [5], using input and output erasures. (Figure 5 depicts the trellis representation of the Hamming code.) The Hamming decoder can alternatively be a decoder that uses the generalized bounded distance decoder, capable of handling erasures.

During the first iteration, the AZD output sequence is decoded by the inner decoder, then passed to the inverse permutation, the outer decoder and the permutation (Figure 4). At the end of the first iteration, the original AZD output sequence is modified by the "error/erasure corrector", which incorporates the corrections made in the first path. The second iteration

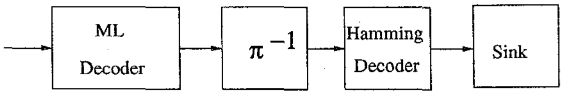

Figure 3: PRML based receiver.

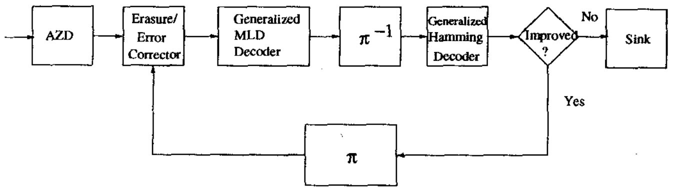

Figure 4: Proposed AZD based iterative receiver.

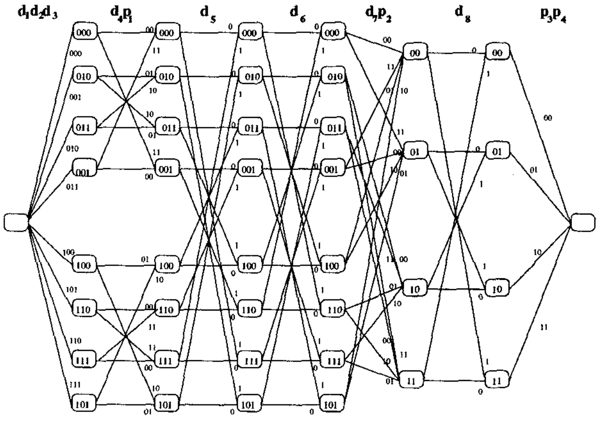

Figure 5: Trellis representation for the shortened (12,8) Hamming code.

applies to this modified AZD output, which revisits the receiver blocks in the same order as in the first iteration. This cyclical decoding procedure repeats itself.

In each iteration, some of the remaining errors and/or erasures will be resolved, and the "error/erasure corrector" modifies the AZD output sequence, by using a simple logic circuit (or logic table) which substitutes some digits of the AZD sequence by their corrected values. In the first iteration, the "error/erasure corrector" plays no role, since the feed-back loop does not provide any information when the iterative decoding just begins.

The iterative procedure should end when all erasures are resolved and no errors are detected, or when no further resolution of error/erasure is achieved, or after a prescribed maximum number of iterations is reached.

The simulation results are shown in Figure 6. For the desired 54 % packet throughput for one packet transmission attempt (99.9 % throughput overall), the iterative receiver outperforms the bit-by-bit detection based receiver by approximately 3 dB and the PRML receiver based by about 1 dB. In most cases, it takes 3-5 iterations to finish the iterative decoding.

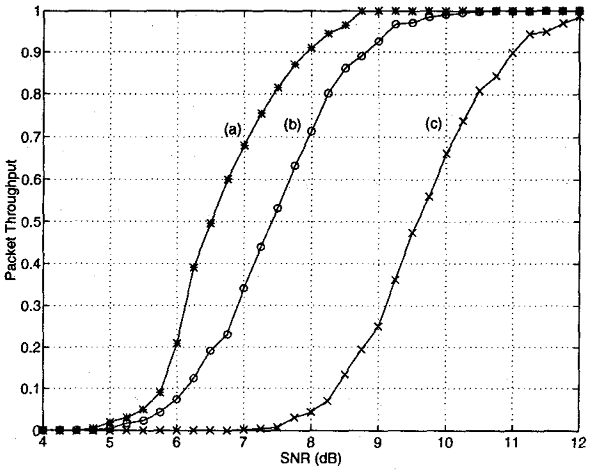

Figure 6: Performance curves for three different receivers (left to right): (a) Proposed iterative AZD based receiver. (b) One-path receiver with soft input maximum likelihood demodulator for duobinary signal. (c) One-path receiver with bit-by-bit duobinary demodulator.

5 Further Extension

In the previous sections we mostly discussed cases where two encoders are concatenated in series. In general, however, we may connect more than two codes and their connecting patterns need not be in series. Parallel as well as serial connections, and a combination thereof, may be a possibility. In fact, recently invented Turbo codes [3] can be viewed as a parallel version of such generalized concatenated codes.

Based on the insights gained in the present study, we propose an explanation for the good characteristics reported on Turbo codes.

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