HIGH-PERFORMANCE SOVA DECODING FOR TURBO CODES OVER cdma2000 MOBILE RADIO

Duanyi Wang
Hisashi Kobayashi

Department of Electrical Engineering, Princeton University
Princeton, New Jersey 08544, USA

Tel: (609) 258-4643 Fax: (609) 258-2158
Email: {duanyi, hisashi}@ee.princeton.edu

ABSTRACT

An improved version of the soft-output Viterbi algorithm (SOVA), which has been theoretically proven to be equivalent to the Max-Log-maximum a posteriori (Max-Log-MAP) algorithm but with greatly reduced decoding complexity, is applied to turbo codes over the 3rd generation cdma2000 mobile radio link. The BER and FER performances of the improved SOVA are evaluated by computer simulation, and compared with the Max-Log-MAP and the conventional SOVA algorithms. The simulation results verify that the improved SOVA scheme can offer the performance comparable to the Max-Log-MAP scheme, while the conventional SOVA is approximately 0.4dB inferior to the Max-Log-MAP at 10^{-3} BER.

I. INTRODUCTION

Turbo codes, introduced by Berrou *et al.* [1], have enjoyed a great attention in recent years. Their powerful error correcting capability is very attractive for mobile wireless applications to combat channel fading. Turbo codes will be adopted as the channel coding schemes for the services of high transmission rates in a number of the 3rd generation IMT2000 mobile systems, cdma2000 being one of them [2].

Among the various soft-output algorithms known today that can provide reliability information together with each decoded bit and so are suitable for the iterative decoding of turbo codes, the maximum *a posteriori* (MAP) algorithm, based on the BCJR algorithm [3], has proven to provide the best overall error performance. However, the processing cost of this algorithm remains a major barrier in its practical implementation. To reduce the processing cost, some sub-optimum soft-output approaches, in which the bit error performance is traded for a reduction in decoding complexity, have been proposed, such as the SOVA (soft-output Viterbi algorithm) and the Max-Log-MAP algorithms [4,5]. The SOVA algorithm, which performs the same operations as the Viterbi algorithm only with additional real value additions and storages for delivery of the soft reliability information, is well known for its much less computational complexity than that for the MAP algorithm. However, the original SOVA suffers from a serious performance degradation. The Max-Log-MAP algorithm, based on the same decoding operation as the original MAP algorithm but operating in the logarithmic domain, outperforms the SOVA by few tenths of a decibel but retains a larger complexity [5].

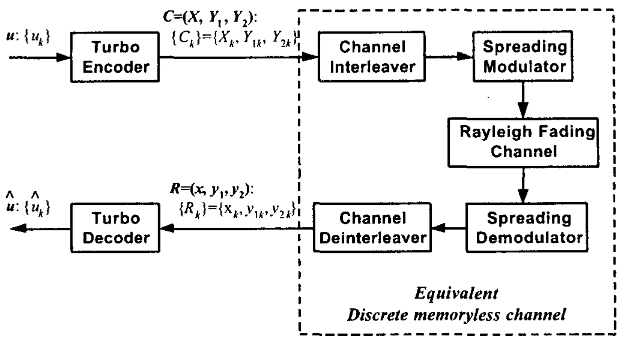
More recently, an improved version of SOVA has been presented and theoretically proven to be equivalent to the Max-Log-MAP algorithm [6]. Such improvement allows the Max-Log-MAP algorithm to be implemented by simply modifying the conventional Viterbi algorithm. It therefore provides an attractive solution to achieve near-optimum soft-input soft-output decoding with lower complexity.

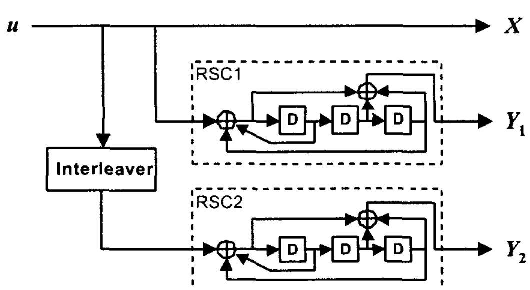
In this paper, we investigate an application of this improved SOVA algorithm to turbo codes to be used in the 3rd generation cdma2000 mobile system. We evaluate the BER (bit error rate) and FER (frame error rate) performances of the improved SOVA, the Max-Log-MAP and the conventional SOVA algorithms by computer simulation. The simulation results verify that the improved SOVA scheme can offer the performance comparable to the Max-Log-MAP scheme, while the conventional SOVA suffers about 0.4dB degradation compared to Max-Log-MAP at 10^{-3} BER.

II. SYSTEM MODEL AND NOTATION

Consider the transmission system model of Figure 1. A rate 1/3 turbo encoder, as specified in the cdma2000 standard [2], is adopted, which is constructed by parallel concatenation of two identical RSC (recursive systematic convolutional) constituent encoders with a turbo internal interleaver in-between. Each RSC encoder has a rate 1/2 with constraint length $K = 4$ (i.e., memory $\nu = 3$) and the parity polynomial $g_1(D) = 1+D^2+D^3$ and the feedback polynomial $g_0(D) = 1+D+D^3$, as shown in Figure 2.

For each input information sequence block $\boldsymbol{u} = \{u_1, u_2, \cdots, u_N\}$ of length N, $u_k \in \{0, 1\}$ for $k = 1, 2, \cdots, N$, RSC1 operates directly on it and produces the first parity sequence $Y_1 = \{Y_{11}, Y_{12}, \ldots, Y_{1N}\}$; RSC2 operates on the interleaved version of u and produces the second parity sequence $Y_2 = \{Y_{21}, Y_{22}, \ldots, Y_{2N}\}$. The resultant overall turbo-coded sequence $C = \{C_1, C_2, \cdots, C_N\}$ is the sum of the three components X, Y_1 and Y_2, i.e., $C = (X, Y_1, Y_2)$. Then the encoded sequence is transmitted over a fading channel, during which the channel interleaving, spreading and a QPSK/BPSK modulation are employed (here we consider the QPSK scheme for the downlink channel). At the receiver side, the sequence applied to the turbo decoder is denoted $R = \{R_1, R_2, \cdots, R_N\}$, where $R = (x, y_1, y_2)$ and





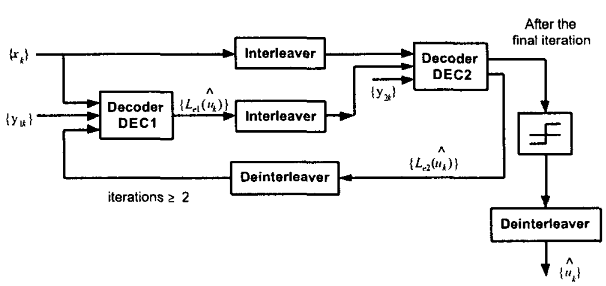


Figure 3. Turbo decoder

$R_k = (x_k, y_{1k}, y_{2k})$ is the noise corrupted version of C_k at time k (assume sufficient channel interleaving).

The global turbo decoder structure, as shown in Figure 3, includes two constituent decoders, DEC1 and DEC2, implementing *a posteriori* probability, and interleavers/deinterleavers with the same interleaving law used in the encoder. The improved SOVA, the Max-Log-MAP and the conventional SOVA algorithms are applied to the constituent decoders respectively in this paper. There are three types of soft inputs to each constituent decoder (DEC1 or DEC2): x, y_i (i = 1 or 2), and the *a priori* information, which is the extrinsic information provided by the other constituent decoder from the previous step of decoding process. The soft output generated by each constituent decoder at time k also consists of three components: a weighted version of x_k, the *a priori* value (i.e., the previous extrinsic information) and a newly generated extrinsic information, which is then provided to the other constituent decoder as *a priori* information for the next step of decoding. Such iterative steps will continue with ever-updating extrinsic information to be exchanged between two decoders until a reliable hard decision can be made.

Let the state of each RSC encoder at time k be s_k, $s_k \in \{0, 1, \ldots, 2^{\nu}-1\}$. The state transition from s_{k-1} to s_k in the trellis corresponds to the k-th information bit u_k. Let the soft-output provided by the constituent decoder for each decoded bit u_k be $\Lambda(u_k)$. When the SOVA-type algorithms are adopted for each constituent decoder, the $\Lambda(u_k)$ is represented by

$$\Lambda(u_k) = \hat{u}_k \hat{L}_k \ , \qquad (1)$$

where $u_k \in \{\pm 1\}$, $\hat{L}_k$ is the soft reliability measure associated with the path decisions.

When the Max-Log-MAP algorithm is adopted, the $\Lambda(u_k)$ is approximated by

$$\begin{aligned}\Lambda(u_k) \approx & \max_{s_k, s_{k-1}}\left(\bar{\gamma}_1(R_k, s_{k-1}, s_k) + \bar{\alpha}_{k-1}(s_{s-1}) + \bar{\beta}_k(s_k)\right) \\ & - \max_{s_k, s_{k-1}}\left(\bar{\gamma}_0(R_k, s_{k-1}, s_k) + \bar{\alpha}_{k-1}(s_{s-1}) + \bar{\beta}_k(s_k)\right)\end{aligned} \qquad (2)$$

where $\bar{\gamma}_i(R_k, s_{k-1}, s_k) = \ln \gamma_i(R_k, s_{k-1}, s_k)$, i = 0, 1, $\bar{\alpha}_k(s_k) = \ln \alpha_k(s_k)$, $\bar{\beta}_k(s_k) = \ln \beta_k(s_k)$. And $\bar{\alpha}_k(s_k)$ and $\bar{\beta}_k(s_k)$ are recursively obtainable by

$$\bar{\alpha}_k(s_k) = \max_{s_{k-1}, i}\left(\bar{\gamma}_i(R_k, s_{k-1}, s_k) + \bar{\alpha}_{k-1}(s_{k-1})\right) \qquad (3)$$

$$\bar{\beta}_k(s_k) = \max_{s_{k+1}, i}\left(\bar{\gamma}_i(R_{k+1}, s_k, s_{k+1}) + \bar{\beta}_{k+1}(s_{k+1})\right) , \qquad (4)$$

and

$$\gamma_i(R_k, s_{k-1}, s_k) = P_r\{u_k = i, s_k \mid s_{k-1}\} \qquad (5)$$

is the branch transition probability, which is computed based on the transition probabilities of the channel and RSC encoders.

For the turbo decoder in Figure 3, $\Lambda(u_k)$ can also be decomposed into the following three terms:

$$\begin{aligned}\Lambda(u_k) &= L_c x_k + L_a(\hat{u}_k) + L_e(\hat{u}_k) \\ &= L_c x_k + L_{e,in}(\hat{u}_k) + L_{e,out}(\hat{u}_k)\end{aligned} \tag{6}$$

where L_c is the channel reliability values. For a Rayleigh fading channel, $L_c = \dfrac{2a_k}{\sigma^2}$, where a_k is the instantaneous fading amplitude with a Rayleigh pdf $p_A(a_k) = 2a_k e^{-a_k^2}$ for $a_k > 0$. The term $L_a(\hat{u}_k)$ is the *a priori* information generated by the previous constituent decoder. It is usually set 0 at the beginning of the iterative decoding process. And $L_e(\hat{u}_k)$ is the extrinsic information.

III. THE IMPROVED SOVA

As above mentioned, assume that each transition in the trellis diagram associated with RSC code corresponds to one information bit u_k, so for any state s_k, there are two branches in the trellis entering it with distinct information bits $u_k = 0$ and $u_k = 1$. Let $\Gamma(s_{k-1}^i, s_k)$, i = 1, 2, be the two candidate cumulative metrics corresponding to the two paths terminating at state s_k with transitions from states s_{k-1}^1 and s_{k-1}^2, the SOVA-type algorithm selects a surviving path for the state s_k after comparing the candidate cumulative metrics, providing not only the hard decision estimate $\hat{\boldsymbol{u}}(s_k)$, but also the corresponding reliability measure $\hat{\boldsymbol{L}}(s_k)$ along the survivor.

Let $\Gamma(s_k)$ denotes the cumulative metric associated with the survivor of the state s_k, $s_k \in \{0, 1, \dots, 2^v-1\}$, the improved SOVA algorithm can be formulated by the following procedures:

Initialization:

- $k = 0$;
- $s_0 = 0$;
- $\Gamma(s_0) = 0$, $\Gamma(s_m) = *$, $s_m \neq s_0$;
- $\hat{u}_0(s_k)$ = null, $s_k \in \{0, 1, \dots, 2^v-1\}$;
- $\hat{L}_0(s_0) = 0$, $\hat{L}_0(s_m) = *$, $s_m \neq s_0$.

Storage:

Assume that the improved SOVA makes the final decision $\hat{u}_k$ after a delay δ (decoding depth), δ being large enough so that all 2^v survivor paths have been merged at time k with sufficient high probability. For each state s_k orresponding to the decoding time k, $s_k \in \{0, 1, \dots, 2^v-1\}$, the improved SOVA decoder stores the following parameters associated with the survivor of this state:

- k (time index, modulo δ+1, δ is decoding depth);
- Cumulative metric values: $\Gamma(s_k)$;
- δ+1 most recent hard decision values:

$$\hat{\boldsymbol{u}}(s_k) = \{\hat{u}_{k-\delta}(s_k), \hat{u}_{k-\delta+1}(s_k), \cdots, \hat{u}_k(s_k)\},$$
$$\hat{u}_j \in \{\pm 1\}, j = k-\delta, \cdots, k;$$

- δ+1 corresponding soft reliability measure values:

$$\hat{\boldsymbol{L}}(s_k) = \{\hat{L}_{k-\delta}(s_k), \hat{L}_{k-\delta+1}(s_k), \cdots, \hat{L}_k(s_k)\},$$
$$0 \le \hat{L}_j \le \infty, j = k-\delta, \cdots, k.$$

Recursion:

At the decoding time $k+1$ for each state s_{k+1}, $s_{k+1} \in \{0, 1, \dots, 2^v-1\}$, the improved SOVA do the following recursion operations:

(1) Decide the surviving $\hat{\boldsymbol{u}}(s_{k+1}) = \{\hat{u}_{k-\delta+1}(s_{k+1}), \hat{u}_{k-\delta+2}(s_{k+1}), \cdots, \hat{u}_{k+1}(s_{k+1})\}$:

i) Compute two candidate cumulative metrics:

$$\Gamma(s_k^i, s_{k+1}),\ i = 1, 2,$$

ii) Select the minimum:

$$\Gamma(s_{k+1}) = \min_{i \in \{1,2\}} \{\Gamma(s_k^i, s_{k+1})\}.$$

iii) Decide $\hat{u}_{k+1}(s_{k+1})$ in the survivor $\hat{\boldsymbol{u}}(s_{k+1})$

(2) Compute the surviving $\hat{\boldsymbol{L}}(s_{k+1}) = \{\hat{L}_{k-\delta+1}(s_{k+1}), \hat{L}_{k-\delta+2}(s_{k+1}), \cdots, \hat{L}_{k+1}(s_{k+1})\}$:

i) Compute the cumulative metric difference:

$$\Delta = \max_{i \in \{1,2\}} \{\Gamma(s_k^i, s_{k+1})\} - \min_{i \in \{1,2\}} \{\Gamma(s_k^i, s_{k+1})\}.$$

ii) Set $\hat{L}_{k+1}(s_{k+1}) = \Delta$. (7)

iii) Compute the remaining $\hat{L}_j(s_{k+1}), j = k-\delta+1, \cdots, k$:

Assume that among the two candidate paths terminating at state s_{k+1}, the surviving path is from the previous state s_k^s, and the concurrent path from state s_k^c, then

if $\hat{u}_j(s_k^s) \neq \hat{u}_j(s_k^c)$,

$$\hat{L}_j(s_{k+1}) = \min\{\hat{L}_j(s_k^s), \Delta\}\ ; \tag{8}$$

if $\hat{u}_j(s_k^s) = \hat{u}_j(s_k^c)$, set

$$\hat{L}_j(s_{k+1}) = \min\{\hat{L}_j(s_k^s), \hat{L}_j(s_k^c) + \Delta\}, \tag{9}$$

Sometimes a factor α is used to divide the Δ in order to prevent overflow with increasing SNR when computing above $\hat{L}(s_{k+1})$ using (7), (8) and (9). It usually takes $\alpha = 4 d_{free}(E_b / N_0)$ as in [4] in order to achieve asymptotically $E[\hat{L}_j] = 1$, where d_{free} is the free distance of the code, E_s/N_0 is the signal-to-noise ratio.

(3) Update survivors:

$$\Gamma(s_{k+1}) \rightarrow \Gamma(s_k)$$
$$\hat{\boldsymbol{u}}(s_{k+1}) \rightarrow \hat{\boldsymbol{u}}(s_k)$$
$$\hat{\boldsymbol{L}}(s_{k+1}) \rightarrow \hat{\boldsymbol{L}}(s_k).$$

The key strategy leading to the improved SOVA superior to the original SOVA is that the improved SOVA adds an updating rule for $\hat{u}_j(s_k^s) = \hat{u}_j(s_k^c)$ situation during the procedure for computing some values $\hat{L}_j(s_{k+1})$, $j \in \{k-\delta+1, \cdots, k\}$ of $\hat{\boldsymbol{L}}(s_{k+1})$. The additional updating rule considers the contribution by the reliability values associated with the concurrent path to the updating of the $\hat{L}_j(s_{k+1})$ in $\hat{\boldsymbol{L}}(s_{k+1})$, which was ignored in the original SOVA, and so can provide more accurate soft reliability values, i.e., better estimates of $\Lambda(u_k)$, than the original SOVA, and was also proven to be able to achieve the same decoding as the Max-Log-MAP algorithm.

IV. SIMULATION RESULT

We evaluate the BER (bit error rate) and FER (frame error rate) performances of a turbo code using the improved SOVA, the Max-Log-MAP and the conventional SOVA decoding schemes by computer simulation. The turbo code adopted is the 8-state rate-1/3 $(1, 13/15, 13/15)_{oct}$ code as shown in Figure 2, with a traditional block interleaver of length 1536 bits that also conforms to the cdma2000 standard. The iterative decoding is done with 8 iterations. For the trellis truncation of the two SOVA-type algorithms, referring to the decoding depth used in the conventional Viterbi algorithm, where δ = (5~10)v, we take $\delta = 24$.

Figure 4 is the SOVA-type algorithms flowchart, for decoding one receiving sequence block. The whole simulation results for above three decoding algorithms are shown in Figures 5(a) and 5(b). The simulation results verify that the improved SOVA scheme can offer the performance comparable to the Max-Log-MAP scheme, especially in the situation of BER versus E_b/N_0, while the conventional SOVA suffers around 0.4dB degradation compared to Max-Log-MAP at 10^{-3} BER.

The cause of the performance of the improved SOVA decoder inferior to that of the Max-Log-MAP decoder as shown in Figure 4 is mainly the use of trellis truncation. According to a special mention in [6], the equivalence between the improved SOVA and the Max-Log-MAP decoding algorithms is under the assumption that no finite-length decoding widow is used in the Viterbi-type SOVA decoding. Therefore, the improved SOVA would come closer to the Max-Log-MAP as the size of the decoding window, i.e., the decoding depth, increases.

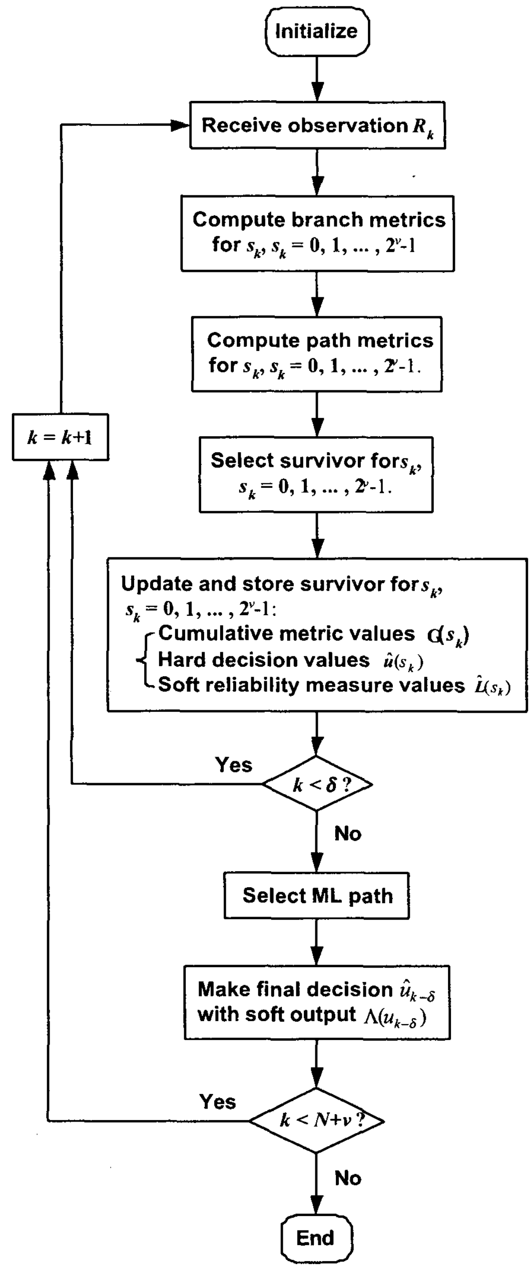
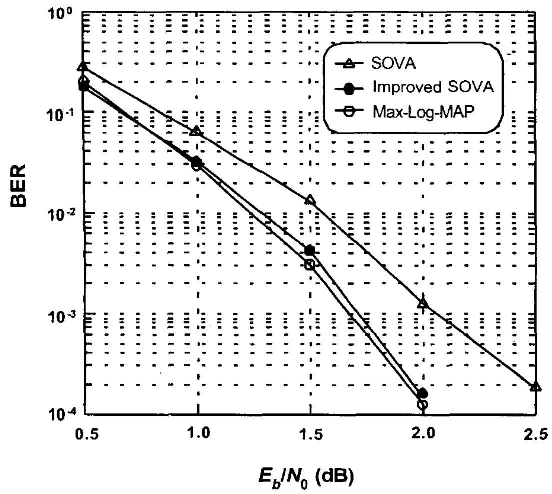
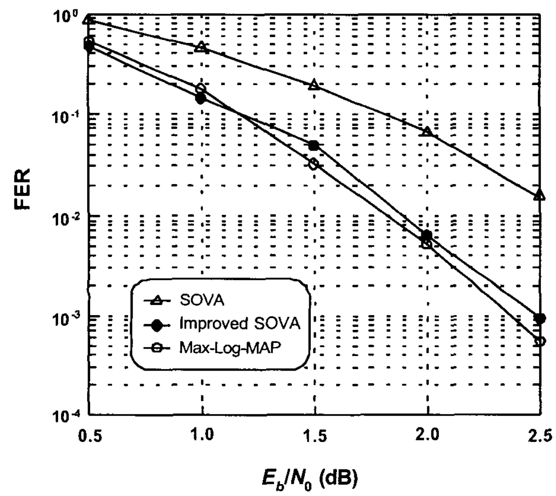


Figure 4. SOVA-type decoding flowchart (for one receiving sequence block)



(a) BER vs E_b/N_0



(b) FER vs E_b/N_0

Figure 5. BER and FER performances of a 8-state rate-1/3 turbo code with three decoding algorithms

V. CONCLUSIONS

In this paper, we have investigated the application of an improved version of SOVA, which was theoretically proven to be identical to the Max-Log-MAP decoding algorithm, to turbo codes for the 3rd generation cdma2000 mobile system. We verify by computer simulation that the improved SOVA decoder can offer performance comparable to the Max-Log-MAP decoder, whereas the conventional SOVA suffers around 0.4dB degradation compared to the Max-Log-Map at 10^{-3} BER. The improved SOVA makes the Max-Log-MAP algorithm implementable by a Viterbi-type trellis decoding approach. It therefore provides an attractive solution to achieve low-complexity near-optimum soft-input soft-output decoding.

The tradeoff between the decoding depth and performance of this improved SOVA is left for further work. Other issues to be addressed include the performances of the improved SOVA applied to different multipath fading environments, the effect of mobile speed and channel estimation and so on.

ACKNOWLEDGMENT

This work has been supported, in part, by the National Science Foundation (NCR-9706045), the New Jersey Center for Wireless Technology (NJCWT), Asahi Chemical Industry Co., the Ogasawara Foundation on Science and Engineering, and Mitsubishi Electric Information Technology Center.

REFERENCES

[1] C. Berrou, A. Glavieux, and P. Thitimasjshima, "Near Shannon limit error-correcting coding and decoding: Turbo-codes(1)," in *Proc. IEEE Int. Conf. on Commun.*, Geneva, Switzerland, May 1993, pp. 1064-1070.

[2] The cdma2000 ITU-R RTT Candidate Submission (TIA/TR45.5), July, 1998.

[3] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. IT-20, pp. 284-287, March 1974.

[4] J. Hagenauer and P. Hoeher, "A Viterbi algorithm with soft-decision outputs and its applications," in *Proc. IEEE Globecom Conf.*, Dallas, TX, USA, Nov. 1989, pp.1680-1686.

[5] P. Robertson, E. Villebrun, and P. Hoeher, "A comparison of optimal and sub-optimal MAP decoding algorithms operating in the log domain," in *Proc. IEEE Int. Conf. on Commun.*, Seattle, WA, USA, June 1995, pp. 1009-1013.

[6] M. P. C. Fossorier, F. Burkert, S. Lin, and J. Hagenauer, "On the equivalence between SOVA and Max-Log-MAP decodings," *IEEE Commun. Lett.*, vol. 2, no. 5, pp. 137-139, May 1998.

[7] P. Zhang and P. Luukkanen, "Performance of turbo codes in 3rd generation cdma2000 mobile system," in *Proc. IEEE Vehicular Technology Conf.*, Amsterdam, The Netherlands, Sept. 1999, vol. 2, pp. 1674 –1677.

[8] N. Guo, F. Khaleghi, A. Gutierrez, J. Li, and M.-H. Fong, "Transmission of high speed data in cdma2000," in *Proc. Wireless Commun. and Networking Conf.*, New Orleans, LA, USA, Sept. 1999, vol. 3, pp. 1442-1445.