Iterative Decoding with Erasures in a Concatenated System with Diversity

Jan Bajcsy and Hisashi Kobayashi
Department of Electrical Engineering
Princeton University
Princeton, NJ 08544, U.S.A.
e-mail: bajcsy@ee.princeton.edu, hisashi@ee.princeton.edu
Fax: (609) 258-3745, Tel: (609) 258-1984

Abstract

We study error correcting codes, which are constructed by using "permutation" and "iterative decoding" in conjunction with "concatenation". These codes include "concatenated codes," "product codes" and recently invented "Turbo codes" as special cases.

We then extend our approach to a continuous channel, by adopting AZD (ambiguity zone detection) at the receiver, which introduces generalized erasures. Iterative decoding can significantly improve the performance of a coded system that contains a channel with memory (e.g., spectral shaping, ISI, multipath delay), modulation with memory (e.g., trellis coded modulation, continuous phase modulation) and/or diversity. Moreover, it is implementable with low decoding complexity and requires a small decoding delay.

Using this framework, we propose two architectures for iterative receivers in an existing wireless packet transmission system. These receivers are fully backward compatible with the existing transmitter, have low decoding complexity and delay, and offer a decoding improvement of several dB's.

1 Introduction

"Concatenation" as a technique to construct powerful codes was first proposed by Forney [5] to connect inner and outer code at the encoder. In many cases an overall optimal decoder may be too complex, and an obvious technique based on one-path decoding in tandem may be far from being optimal.

Recently invented Turbo codes [4] propose a parallel concatenation of two encoders, separated by a long interleaver, that achieves performance close to the Shannon capacity limit. In general, however, we may connect more than two codes and their connecting patterns need not be in series. Parallel as well as serial connections, and a combination thereof, may be a possibility.

An "iterative decoder", which can be shown to provide an approximation of an overall optimal decoder, with low complexity in its implementation, has been studied in [3]. We can further improve the overall structure by inserting a "permutation" between the concatenated encoders. Permutation, a more general concept than interleaving, is beneficial not only for channels with burst noise, but also for channels with random noise, and can be introduced without any degradation in information rates.

The structure of this paper is as follows. Section 2 introduces the background of generalized concatenated systems. The idea of iterative decoding using ambiguity zone detection is presented in Section 3. Section 4 applies this approach to an existing wireless packet transmission system. After formulating this system as a generalized concatenated system, two iterative decoders are proposed. A bound on the maximum performance improvement is given and simulation results are presented. Concluding remarks and further ways to improve the system under consideration are discussed in Section 5.

2 Generalized Concatenated Systems

In the present paper we extend the classical concept of concatenated codes to a larger class of systems. We define a *"generalized concatenated system"* as a system that can be built by concatenation (serial, parallel or combination) of blocks that include some redundancy or memory. Such blocks include not only channel encoders, as in the conventional concatenated codes, but also

1. a channel with spectral shaping (e.g., partial-response coding) or line coding,
2. a channel with ISI (intersymbol interference) and/or multipath delay,
3. modulation with memory (e.g., trellis coded modulation, continuous phase modulation [1])

The concatenation may be a result of a cascade of encoders, time diversity or space diversity at the transmitter. We insert permutations between these building blocks. Hence concatenated codes, product codes and recently introduced Turbo codes [4] are treated as special cases of the generalized concatenated system.

3 New Decoding Technique

We introduce a novel receiver structure [2] which combines AZD (ambiguity zone detection) and iterative decoding. The idea of AZD (or sometimes called the null zone detector) was successfully applied to a partial response system [7]. A continuous channel with AZD at the receiver front can be viewed as a discrete channel with generalized erasures. In our iterative decoding, AZD provides flexibility of deferring decision on unreliable digits by labeling them as "ambiguous digits" or "generalized erasures".

The iterative decoding technique we introduce is, in concept, similar to iterative decoding procedures used in decoding Turbo codes [4] and soft decision decoding scheme discussed by Hagenauer et al. [10] in decoding a concatenated code with Reed-Solomon code and convolutional code. However, the combination of iterative decoding and AZD in the context of the generalized concatenated system is novel to our best knowledge. The iterative decoder makes step-by-step resolutions of these ambiguous digits by capitalizing on the redundancy introduced by the error correcting code, and modulation/channel with memory. The permutation alleviates the clustering effect of residual erasures introduced in the iterative decoding steps.

We present simulation results, which demonstrate a significant performance improvement over previously known approaches such as PRML (partial-response coding, maximum-likelihood decoding). Our iterative decoding technique can be applied to a number of practical systems including digital magnetic recording and wireless communications [2].

4 An Example

In this section we will illustrate our scheme on an existing wireless packet transmission system. We first give the specifications of the system, both at the transmitter and at the receiver side. Then we describe two iterative receivers with AZD (Ambiguity Zone Detection), based on the generalized concatenated systems approach. Performance evaluation of these receivers on the AWGN channel follows. A comparison is given with the performance of the original receiver and of a standard PRML receiver.

4.1 System Specification

The current transmitter is depicted as a discrete baseband system in Figure 1. Transmission of each packet proceeds as follows. A packet contains 160 information bits which are first encoded the by an error correcting code (ECC) – a shortened Hamming (12, 8) code with generator matrix

$$\mathbf{G} = \begin{bmatrix} 1&0&0&0&0&0&0&0&1&1&0&0 \\ 0&1&0&0&0&0&0&0&1&0&1&0 \\ 0&0&1&0&0&0&0&0&0&1&1&0 \\ 0&0&0&1&0&0&0&0&1&1&1&0 \\ 0&0&0&0&1&0&0&0&1&0&0&1 \\ 0&0&0&0&0&1&0&0&0&1&0&1 \\ 0&0&0&0&0&0&1&0&1&1&0&1 \\ 0&0&0&0&0&0&0&1&0&0&1&1 \end{bmatrix} . \qquad (1)$$

Resulting 240 bits are permuted by a 20 × 12 block interleaver and then passed to the duobinary modulator that includes both precoding [8] and actual duobinary modulation ($g(D) = 1 + D$) in discrete time. Its representation by a trellis diagram is in Figure 3(a). To achieve sufficiently high packet throughput, space and time diversity are used. Each packet is transmitted from three different locations, three different times from each location, i.e., each packet is transmitted nine times overall.

The channel noise is assumed to be additive white Gaussian noise (AWGN) in the baseband. The desired quality of service at the receiver is 99.9 % packet throughput or about 54 % packet throughput in each independent transmission attempt.

The current system uses a receiver shown in Figure 2(a). It is based on bit-by-bit detection of the duobinary signal and a syndrome decoder of the (12, 8) code. Nine independent trials for successful decoding of each packet are made by the receiver.

4.2 Proposed Iterative Receiver

The proposed iterative receiver is depicted in Figure 2(c). The basic idea behind this receiver is that two concatenated decoders working in tandem are helping one another to correct more errors and remove erasures as the decoding proceeds during each reception.

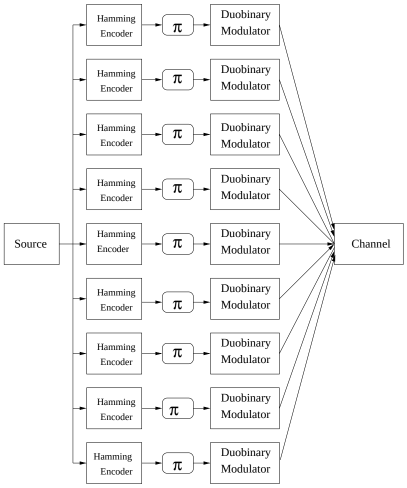

Figure 1: Existing transmitter from Section 4 including time and space diversity.

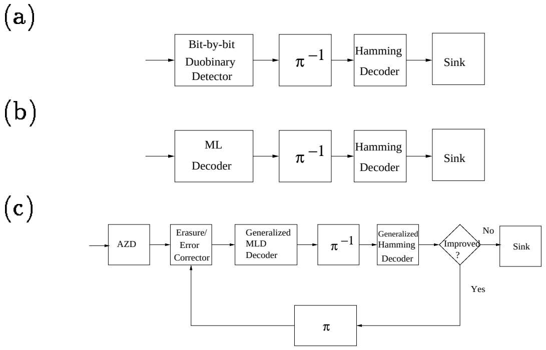

Figure 2: Block diagrams of the three receivers considered in Section 4.2 (a) Current bit-by-bit detection based receiver (b) PRML based receiver. (c) Proposed AZD based iterative receiver.

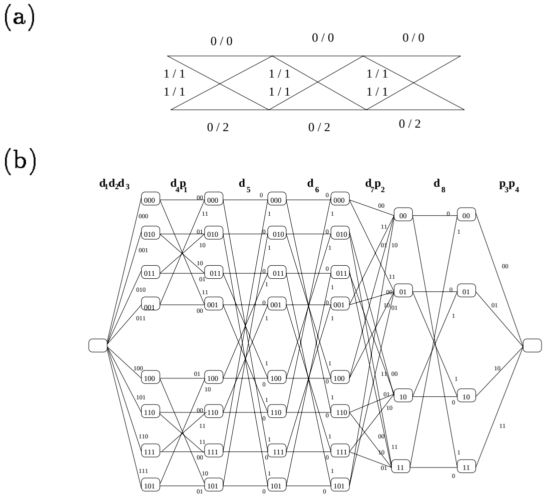

Figure 3: Trellis representation of the (a) duobinary signaling, (b) shortened (12,8) Hamming code.

The receiver functions as follows. First, the channel output symbols are passed through an ambiguity zone detector, i.e., a threshold detector that quantizes the data to five levels at the receiver front, as discussed in [7]. Thus in addition to the duobinary symbols 0, 1 and 2, the data passed to the inner decoder contains erasures. The concatenated decoders for the duobinary code and Hamming code form a loop. They are separated by the permutation (in the feed-back path) and its inverse-permutation (in the forward path) to preserve the order of the data. These two decoders are capable of performing decoding with erasures. The decoders can be based on soft decision maximum likelihood decoders [6], using input and output erasures. (Figure 3(b) depicts the trellis representation of the Hamming code.) The Hamming decoder can alternatively be a decoder that uses the generalized bounded distance decoder, capable of handling erasures.

During the first iteration, the AZD output sequence is decoded by the inner decoder, then passed to the inverse permutation, the outer decoder and the permutation (Figure 2(c)). At the end of the first iteration, the original AZD output sequence is modified by the "error/erasure corrector", which incorporates the corrections made in the first path. The second iteration applies to this modified AZD output, which revisits the receiver blocks in the same order as in the first iteration. This cyclical decoding procedure repeats itself.

In each iteration, some of the remaining errors and/or erasures will be resolved, and the "error/erasure corrector" modifies the AZD output sequence, by using a simple logic circuit (or logic table) which substitutes some digits of the AZD sequence by their corrected values. In the first iteration, the "error/erasure corrector" plays no role, since the feedback loop does not provide any information when the iterative decoding just begins.

The iterative procedure should end when all erasures are resolved and no errors are detected, or when no further resolution of error/erasure is achieved, or after a prescribed maximum number of iterations is reached.

The simulation was performed on the AWGN channel. The threshold values A and B of the AZD detector are depicted in Figure 4. Their optimum values were calculated as a function of SNR by maximizing the capacity of the resulting discrete memoryless channel. Nonetheless, the simulation results show that the performance is robust to their small changes in the SNR region of interest and a reasonable choice is A=0.4, B=0.65.

The simulation results are shown in Figure 5. To make a fair comparison, we also considered a receiver

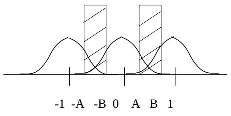

Figure 4: Ambiguity zone detection zones for the duobinary signal.

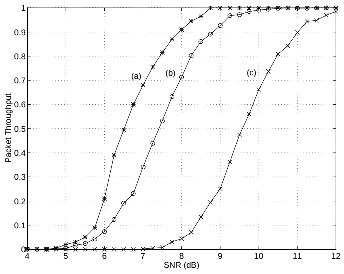

Figure 5: Performance curves for three different receivers (left to right): (a) Proposed iterative AZD based receiver, (b) PRML based receiver, (c) One path receiver with bit-by-bit duobinary demodulator.

based on maximum likelihood detection (PRML) followed by a syndrome decoder of the Hamming code (Figure 2(b)). For the desired 54 % packet throughput for one packet transmission attempt (99.9 % throughput overall), the iterative receiver (curve (a)) outperforms the bit-by-bit detection based receiver by approximately 3 dB and the PRML receiver based by about 1 dB. In most cases, it takes 3-5 iterations to finish the iterative decoding.

4.3 Iterative Receiver Using Diversity

The transmitted packet of duobinary symbols is denoted as a sequence $x[n]$ $(n = 1, 2, ..., 240)$, and the average symbol energy is E_s. We denote $y_1[n]; y_2[n]; ...; y_9[n]$ the 9 received copies of the packet from the channel with

$$y_i[n] = x[n] + z_i[n], \tag{2}$$

where $z_i[n]$ are assumed to be i.i.d. normal random variables $N(0, \sigma^2)$. The channel signal to noise ratio (SNR) is then $\frac{E_s}{\sigma^2}$.

The optimal receiver taking into account the diversity would first obtain all 9 results of transmissions of a given packet $y_1[n], y_2[n], ..., y_9[n]$. It would decode their average value $y[n]$

$$y[n] = \frac{1}{9}\sum_{i=1}^{9} y_i[n] = x[n] + z[n], \tag{3}$$

where $z[n]$ are i.i.d., normal $N(0, \frac{\sigma^2}{9})$. Consequently, the SNR observed by the decoder becomes nine times lower than the channel SNR, i.e., $\frac{9E_s}{\sigma^2}$. This means an improvement by 9.5 dB over the channel SNR. Consequently, if we used the iterative receiver proposed in Sections 4.2., its performance curve (a) from Figure 5 will get shifted by 9.5 dB to the left, hence achieving 99.9 % packet throughput at about 0.5 dB. Consequently, the decoding gain over the existing receiver would be 9.25 dB. Some word of caution is needed here, with respect to this number:

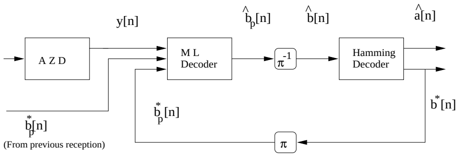

Figure 6: Iterative decoder utilizing diversity.

1. Time delay: The optimal time diversity handling receiver analyzed here would introduce too much of a delay. We would have to wait until the last reception to start decoding even for reliably received packets.
2. Model validity: The result is valid up to the model validity. If the noise is not i.i.d. (e.g., strongly data dependent) the improvement over channel SNR may be smaller than 9.5 dB.

Nonetheless, the analysis shows the potential a significant improvement over currently used receiver. Consequently, a practical decoding scheme has to be designed in order to exploit as much of this potential.

We propose an architecture of a receiver that exploits the diversity, brings no time delay in reception of good packets and has almost no extra storage needs. This receiver proceeds as follows:

1. Wait for a new reception of a packet that has not been completely decoded before.
2. Using successfully decoded bits from previous receptions of the packet as well as the channel data, try to decode the packet.
3. If the packet has been decoded successfully, deliver it to the sink. Otherwise store all reliably decoded bits, label the rest as erasures and go to step 1.

Figure 6 shows the block diagram of this decoder. Note that during the first iteration, the ML duobinary decoder proceeds using the channel data as well as the bits decoded successfully in previous receptions. In the following iterations it uses also the data from the feedback loop.

Initial simulation results on the AWGN channel show (Figure 7) that the *overall* throughput 99.9 % (or loss of 10^{-3}) is achieved at channel SNR about 3.7 dB (i.e., more than 6 dB improvement over the current

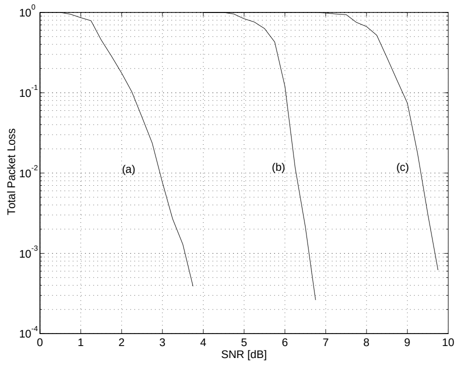

Figure 7: Overall packet loss for three different receivers (left to right): (a) Iterative AZD based receiver that exploits diversity. (b) Iterative AZD based receiver from Section 4.2 (c) One path receiver with bit-by-bit duobinary. demodulator.

receiver). For comparison, the performance curves for the original receiver and the first proposed iterative receiver are given.

5 Conclusion and Future Work

This paper first discusses the generalized concatenated systems, i.e., digital communications or recording systems which contain two or more connected encoders. Their connecting pattern can be in series, in parallel and a combination thereof. In fact, recently invented Turbo codes [4] can be viewed as a parallel version of such generalized concatenated codes. A novel receiver structure [2] which combines AZD (ambiguity zone detection) and iterative decoding, is then reviewed for these systems.

This approach is then applied to an existing wireless packet transmission system. As sections 4.2 and 4.3 shows, there is a potential for a significant decoding gain at the receiver, as compared to the current receiver. The goal is to utilize most of this potential by using the invented iterative decoding scheme with generalized erasures. Two iterative receivers are proposed and as the simulation results show, they achieve an improvement of several dB over the current receiver.

Further improvements in the performance of the wireless packet transmission system are possible by modifying the transmitter side. In the present system, the same message is sent nine times, i.e., three times from three different transmitters. Although this simple diversity scheme is easy to implement, a lot of worthy information is being wasted by sending the same data. A better strategy would be to send each time differently permuted version of the encoded message. The proposed iterative decoding is also suited to receiving messages in this format.

The current error correcting code is a shortened Hamming code. Although it is easy to implement at both encoding and decoding sides, it is not as powerful as some more advanced codes such as BCH codes, Reed-Solomon codes or product codes. Hence, it would be worthwhile to explore the use of a more powerful code at the transmitter.

Acknowledgements: The present work has been supported, in part, by the National Science Foundation, DoD/FRI, ARPA/Hanscom, Seiko Communications Systems, Inc., and the Ogasawara Foundation for the Promotion of Science and Engineering.

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