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A Decision-feedback Receiver for Channels with Strong Intersymbol Interference

Abstract: This paper deals with the problem of equalizing channels containing strong intersymbol interference. Typical of such channels are those of digital magnetic recording systems and data communication systems with partial-response signaling. First we discuss reasons that a conventional receiver with a linear equalizer cannot efficiently compensate for distortion in such channels. We then present a new receiver configuration in which the equalizer and quantizer are embedded in an inverse filter circuit that eliminates major intersymbol interference components. This configuration allows us to use a simple iteration algorithm to adaptively adjust the equalizer. Application of the scheme to digital magnetic recording data is discussed as an illustrative example.

1. Introduction

It is generally known in the field of communication technology that the maximum rate at which digital data can be successfully transmitted through a band-limited channel depends more upon the effects of intersymbol interference than on channel noise [1]. An engineering problem of essentially the same nature arises in digital magnetic recording systems, in which the maximum recording density is determined to a great extent by intersymbol interference introduced via a space-limited reading head [2]. Various coding schemes for reducing permissible time spacing in data transmission and for increasing the packing of data on recording media have recently been reviewed by Kobayashi [3] and Cullum [4].

One of the frequently adopted techniques in high-speed data transmission involves "correlative-level coding" [5] or "partial-response" signaling [6], in which a controlled amount of intersymbol interference is introduced deliberately so that the overall system becomes insensitive to imperfections of the channel characteristic [7]. We have shown earlier [8] that in digital magnetic recording systems the "differentiation operation" inherent in the read-back process (i.e., the induced voltage is proportional to the time-derivative of magnetization) creates in effect a correlative-level coded sequence. Optimal decision algorithms for decoding digital sequences distorted by such strong (but known) intersymbol interference have been recently discussed by the present authors [8,11], Kobayashi [9,10], Forney [12,13], Ungerboeck [14], and others.

In practice, however, digital sequences may be distorted by unknown, time-varying intersymbol interference components that are more significant than random noise in limiting data transmission rates and recording density. Conventional adaptive equalizers [1,15,16] could eliminate such interference components effectively if it were not for the simultaneous presence of strong intersymbol interference components deliberately inserted, such as those in partial-response systems. To the authors' knowledge, no adaptive equalizers that can operate satisfactorily under such circumstances have been reported previously. This paper introduces a decision-feedback receiver to serve for that purpose.

In Section 2 we present a theoretical investigation into the effectiveness of the standard linear equalizer as a means of compensating for distortion in a channel with strong intersymbol interference. The difficulties in attempting to use a standard equalizer for this purpose are identified. In Section 3 we discuss a new adaptive receiver appropriate to essentially any type of linear channel with intersymbol interference. This scheme is an extension of several receiver structures reported in our earlier work on error detection [8] and correction [11] in partial-response channel systems. Section 4 describes how the new structure can be applied in a digital magnetic recording system to increase recording density.

2. Difficulties of the conventional linear equalizer in the presence of strong intersymbol interference

Throughout this paper we deal with a linear discrete system representation as depicted in Fig. 1. The channel is treated as a digital link comprising a signal generator, modulator, transmission medium, demodulator, filter, and sampler. This schematic representation includes a

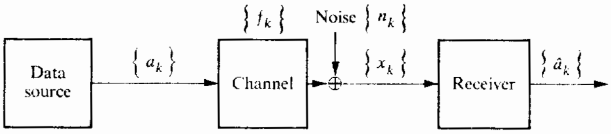

Figure 1 Discrete linear system.

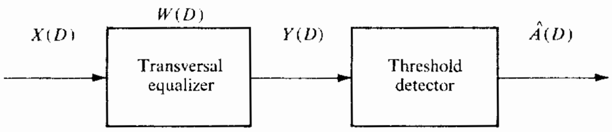

Figure 2 Conventional receiver structure.

class of partial-response or correlative-level coding systems: Such a signaling scheme can be realized by shaping any part of the channel just described. Here the data sequence $\{a_k\}$ takes on integer values $0, 1, 2, \cdots, m-1$, and $\{f_k\}$ represents the impulse response sequence of the channel. The channel output sequence $\{x_k\}$ including the effect of additive noise $\{n_k\}$ is given by

$$x_k = \sum_j f_j a_{k-j} + n_k. \tag{1}$$

In a conventional receiver (Fig. 2) with a linear equalizer, the sequence $\{x_k\}$ is first passed into a transversal-filter equalizer with tap coefficients $\mathbf{W} = \{W_i\}$, $-N_1 \le i \le N_2$, where N_1 is the number of taps preceding the central tap W_0 and N_2 is the number of taps succeeding W_0. The equalizer $\mathbf{W}$ is designed to approximate the inverse filter of the channel response $\{f_k\}$. Hence, the following power-series (or generating function) expression in terms of delay operator D should hold:

$$H(D) \equiv F(D)W(D) \approx 1, \tag{2}$$

where

$$F(D) = \sum_{k=-\infty}^{\infty} f_k D^k \tag{3}$$

and

$$W(D) = \sum_{i=-N_1}^{N_2} W_i D^i. \tag{4}$$

One of the most frequently adopted criteria for equalization is minimization of the peak distortion [1,15] defined by

$$D_h = \frac{1}{h_0} \sum{}' |h_k|, \tag{5}$$

where $\{h_k\}$ is the response function observed at the equalizer output as defined by (2), and $\sum'$ means summation over all k excluding the term $k = 0$. Lucky [15] has shown that if the distortion prior to equalization is less than unity, i.e., if

$$D_f = \frac{1}{f_0} \sum{}' |f_k| < 1, \tag{6}$$

the minimization of D_h can be attained by a simple iterative method. However, if $D_f > 1$, there exists no known iterative algorithm that assures convergence of $\{W_i\}$ to the optimal value.

Another criterion often used is the minimization of the mean-squared distortion [1], which is defined by

$$E_h^2 = \frac{1}{h_0^2} \sum{}' h_k^2. \tag{7}$$

This criterion does not require a condition like (6) to assure convergence to the solution, but when the initial distortion E_f^2 is heavy, the convergence rate is slow and the signal-to-noise ratio at the equalizer output will become degraded. In the rest of the present section we elaborate on the convergence rate in some detail, since complete understanding of the difficulties inherent in these channels will lead to the new scheme discussed in Section 3.

In the actual transmission of data we cannot observe the isolated channel response $\{h_k\}$. So, in general, we estimate $\{h_k\}$ by cross-correlating the received, noisy sequence and its decision output. If we assume that the data sequence $\{a_k\}$ is uncorrelated and the noise $\{n_k\}$ has zero mean and is independent of $\{a_k\}$, then minimization E_h^2 is, practically speaking, equivalent to minimization of the squared-error sum

$$P = \sum_{k \in K} (y_k - a_k)^2, \tag{8}$$

if the number of data points in the observation period K is sufficiently large. Here $\{y_k\}$ is the equalizer output; i.e.,

$$Y(D) = W(D)X(D). \tag{9}$$

With a straightforward manipulation, P can be expressed in a quadratic form of the N-dimensional vector $\mathbf{W}$, where $N = N_1 + N_2 + 1$ [17].

$$P = \langle \mathbf{W}, \mathbf{R}\,\mathbf{W} \rangle - 2\,\langle \mathbf{W}, \mathbf{b} \rangle + \sum_{k \in K} a_k^2, \tag{10}$$

where $\langle\ ,\ \rangle$ denotes the inner-product, $\mathbf{R}$ is an $N \times N$ matrix whose elements are defined by

$$R_{nn'} = \sum_{k \in K} x_{k-n} x_{k-n'}, \qquad -N_1 \le n,\ n' \le N_2 \tag{11}$$

and $\mathbf{b}$ is an $N \times 1$ matrix:

$$b_n = \sum_{k \in K} x_{k-n} a_k, \qquad -N_1 \le n \le N_2. \tag{12}$$

Clearly, minimization of P is attained when $\mathbf{W}$ satisfies the linear equation

$$\mathbf{R}\,\mathbf{W} = \mathbf{b}. \tag{13}$$

Therefore the equalizer can simply be a special purpose hardware device that solves (13) in some iterative way. For details of such algorithms and their implementations the reader is referred to [1,15,16,17].

The problem here is equivalent to inverting a positive-definite matrix **R** by some iterative method. The rate of convergence to the optimal solution is highly dependent on the maximum and minimum eigenvalues $\mu_{\max}$ and $\mu_{\min}$, of the matrix **R**. If the iteration is done by a simple gradient method then P_i, the squared-error sum after the ith iteration, converges at least as fast as the geometric progression given by [16,17]

$$P_i - P^* \leq \rho^i (P_0 - P^*), \tag{14}$$

where

$$\rho = \frac{\mu_{\max} - \mu_{\min}}{\mu_{\max} + \mu_{\min}} \leq 1, \tag{15}$$

P_0 is the initial distortion and P^* is the attainable minimum P.

It is known [17] that both the iterative method based on the steepest-decent technique with optimal step size and the conjugate-gradient method provide faster convergence, and that P_i is bounded by

$$P_i - P^* \leq \rho^{2i} (P_0 - P^*). \tag{16}$$

It is also known from the theory of spectral analysis [18] that eigenvalues of **R** are closely related to the power spectrum $P_x(\omega)$ of the corresponding random sequence, i.e., all eigenvalues are bounded by the maximum and minimum of $P_x(\omega)$:

$$\max_\omega P_x(\omega) \geq \mu_{\max} \geq \mu_{\min} \geq \min_\omega P_x(\omega). \tag{17}$$

Furthermore, it is known that as N, the size of **R**, goes to infinity, the maximum and minimum eigenvalues attain the bounds given above [18]. It is not difficult to see that $P_x(\omega)$ is proportional to $|F^*(\omega)|^2$ if the noise is weak or the observation interval K is sufficiently large. Here $F^*(\omega)$ is the Fourier transform of the channel transfer function:

$$F^*(\omega) = \sum_k f_k e^{-ik\omega} = F(e^{-i\omega}), \qquad -\pi \leq \omega \leq \pi. \tag{18}$$

Therefore, if N is sufficiently large, the parameter ρ is given approximately by

$$\rho = \frac{r-1}{r+1}, \tag{19}$$

where

$$r = \frac{\max |F^*(\omega)|^2}{\min |F^*(\omega)|^2} \geq 1. \tag{20}$$

If $|F^*(\omega)|$ is nearly flat, r is close to unity and hence ρ is small, which implies fast convergence. On the other hand, if $|F^*(\omega)|$ contains heavy amplitude distortion, r is large and ρ is close to one; then the convergence may be quite slow.

Many important channels have a null in $F^*(\omega)$: for example, the so-called duobinary system has a zero at $\omega = \pi$, and a digital magnetic recording channel has a zero at $\omega = 0$. As ρ approaches unity, not only does the convergence speed decrease, but also the matrix **R** approaches singularity. The physical effect is increasing instability of the equalizer and great amplification of the noise level. For a more detailed argument about the equalizer and SNR degradation see [19].

As an alternative approach one might attempt to design the equalizer $W(D)$ so that the overall system response is an approximation to some known or desired response function $G(D)$:

$$F(D)\,W(D) \approx G(D). \tag{21}$$

The "reference" response $G(D)$ can be specified by design (as in partial-response signaling) or can be determined on the basis of the observed channel response. In other words, we might consider that $G(D)$ represents the major components of the intersymbol interference contained in $F(D)$. Although this approach looks feasible at first, it has the following two defects. The first defect becomes evident when we rewrite (21) as

$$Q(D)W(D) \approx 1, \tag{22}$$

where

$$Q(D) = F(D)/G(D). \tag{23}$$

It is clear that $W(D)$ should approximate the inverse of $Q(D)$ in the regular sense of equalization. However, $Q(D)$ may contain heavy distortion (i.e., $D_q > 1$) even if $G(D)$ is nearly equal to $F(D)$. In fact, it should be recalled [3] that the partial-response signaling or correlative-level coding is adopted for the purpose of canceling intersymbol interference that would otherwise arise in the "channel proper" $Q(D)$. In other words, we introduce the controlled amount of distortion, $G(D)$, only when the channel already contains too much distortion to allow transmission of the original m-level sequence $A(D)$.

The second difficulty is that the input sequence to the equalizer is no longer $A(D)$, but $C(D) = G(D)A(D)$, which is a correlated sequence. Therefore, a simple algorithm such as Lucky's adaptive equalization method [1,15] is not applicable here. As for the equalization method based on the minimum mean-squared error criterion, the entire argument given earlier remains valid, except that the sequence $\{a_k\}$ of (8) is now replaced by $\{c_k\}$. The problem of slow convergence remains unchanged.

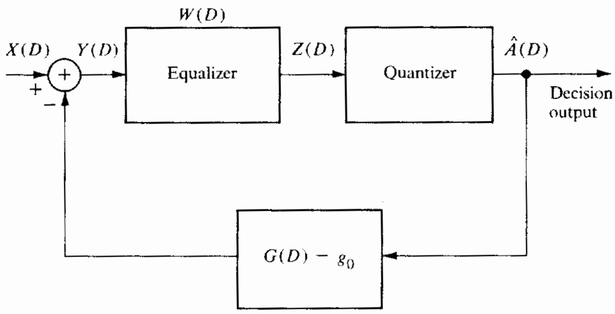

Figure 3 Basic structure of the new receiver.

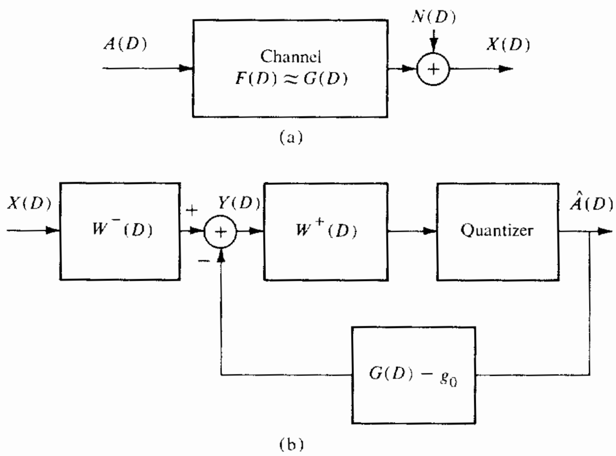

Figure 4 (a) Channel $F(D)$ with strong intersymbol interference $G(D)$; (b) receiver configuration with precursor and main equalizers.

3. A new adaptive receiver structure

The receiver structures examined in Section 2 failed mainly because a linear equalizer was inserted between the channel output and the threshold detector. In this section we present a new receiver structure in which the linear equalizer and quantizer (threshold detector) are embedded in an inverse filter having the transfer function $1/G(D)$, where $G(D)$ represents the main pulse and major intersymbol interference as defined in Section 2:

$$G(D) = \sum_{i=0}^{L} g_i D^i. \tag{24}$$

We do not lose generality by assuming $g_i = 0$, $i = 0$ [8,11].

We now define a response sequence $R(D)$ by

$$R(D) = F(D) - [G(D) - g_0]. \tag{25}$$

By appropriate choice of $G(D)$ we can let $R(D)$ contain a large central peak but remain small elsewhere. We attempt to equalize this "residual" response $R(D)$ by choosing an equalizer $W(D)$ such that

$$W(D)\, R(D) \approx 1. \tag{26}$$

Given a noisy channel output $X(D)$, we now define a sequence $Y(D)$ by

$$Y(D) = X(D) - [G(D) - g_0]A(D). \tag{27}$$

In this case $Y(D)$ is free from most intersymbol interference and the equalizer output is approximately equal to $A(D)$ since

$$\begin{aligned} W(D)Y(D) &\approx W(D)F(D)A(D) \\ &\quad - W(D)[G(D) - g_0]A(D) \\ &= W(D)R(D)A(D) \\ &\approx A(D). \end{aligned} \tag{28}$$

The data in the feed-back loop are free of additive noise, and this property makes this inverse filter stable, even if $1/G(D)$ itself is unstable.

If Lucky's criterion,

$$D_r = \frac{1}{r_0} \sum{}' |r_k| < 1, \tag{29}$$

is satisfied for the residual response $R(D)$, then his simple iterative algorithm to adjust $W(D)$ is applicable since the equalizer input is now the original uncorrelated sequence $A(D)$ (with some intersymbol interference and noise). By substituting the decision output $\hat{A}(D)$ for the $A(D)$ of (27) we can construct an adaptive receiver as shown in Fig. 3.

A problem arising here is that the decision output is obtained with N_1-digit delay, where N_1 is the number of taps preceding the central tap W_0 of the equalizer. It should be clear that N_1 must be less than the delay of $G(D) - g_0$. For example, when $G(D) = 1 - D^2$, N_1 must be less than two. This implies that $W(D)$ cannot, in general, eliminate all precursor (or front-end) intersymbol interference components, which are not negligibly small in vestigial sideband or single sideband transmission.

If we assume that front-end interference terms alone do not constitute heavy distortion (which will be almost always the case unless $G(D)$ is poorly chosen), then this difficulty can be removed by adding a precursor equalizer ahead of the main receiver. Figures 4(a) and (b) depict the total system which combines the channel with major intersymbol interference $G(D)$ and the corresponding new adaptive receiver just described, where $W^-(D)$ represents the precursor equalizer:

$$W^-(D) = \sum_{i=-N_1}^{0} W_i^- D^i, \tag{30}$$

and $W^+(D)$, the main equalizer:

$$W^+(D) = \sum_{i=0}^{N_2} W_i^+ D^i. \tag{31}$$

One may use the precursor equalization algorithm suggested by Sha and Tang [20].

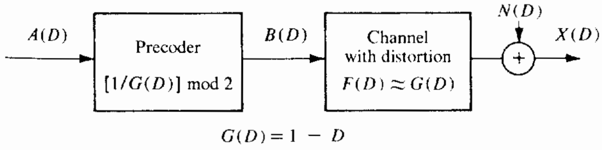

Figure 5 Discrete system representation of a magnetic recording system with NRZI recording method.

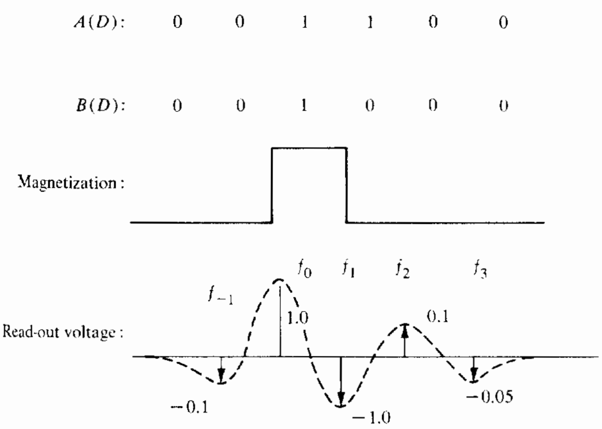

Figure 6 Response function $\{f_k\}$ in a magnetic recording system with NRZI recording.

If we replace $W^+(D)$ by a constant gain factor $1/g_0$ and take out the quantizer in Fig. 4(b), the circuit will become the inverse filter $1/G(D)$. Thus, the new structure is seen to be an extension of the receiver structure discussed by the authors in [8,11].

4. Example: A digital magnetic NRZI recording system

Our earlier study [8] showed that the digital magnetic recording system with NRZ (non-return to zero) recording is equivalent to a linear discrete system with $G(D) = 1 + D$ and the number of signal levels m equal to two. The previous study also showed that the so-called NRZI (non-return to zero, invert) recording system corresponds to a system $G(D) = 1 - D$, $m = 2$, with precoding $[1/G(D)]$ mod 2 (Fig. 5). Precoding is a technique to avoid the propagation of errors [1,5,6,8,11].

Let the unit pulse response [a single pulse in $B(D)$ sequence] be given by $\{f_k\}$ as depicted in Fig. 6. For ease of illustration we assume

$$f_k = 0, \qquad k < -1. \tag{32}$$

Furthermore, we assume that the precursor equalizer $W^-(D)$ has only one unit time delay, i.e., $N_1 = 1$. We denote the response output of the precursor equalizer by $\{h_k\}$. Initially, we set

$$W_0^- = 1, \qquad W_{-1}^- = 0. \tag{33}$$

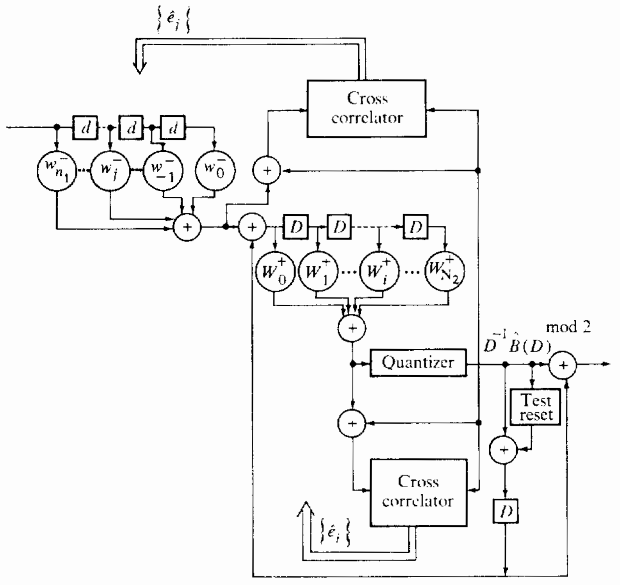

Figure 7 Receiver configuration with adaptive precursor and main equalizers.

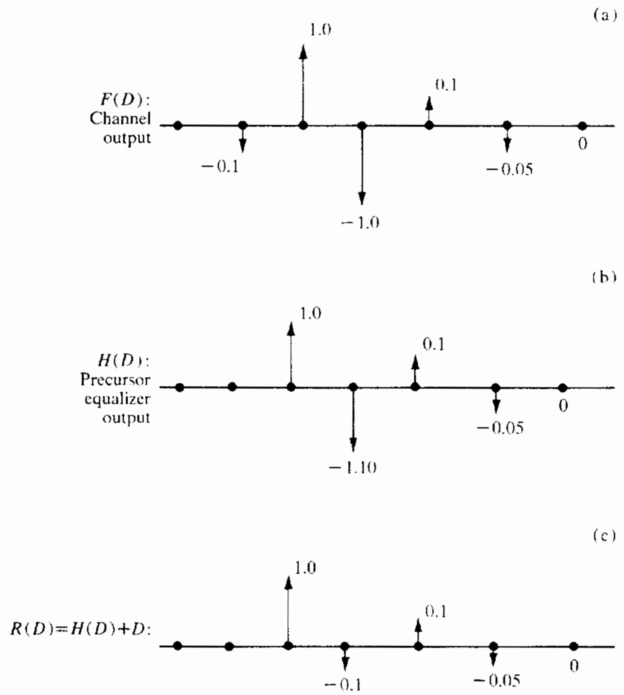

Figure 8 Impulse responses at three stages of the receiver shown in Fig. 7.

Then it is clear that $h_k = f_k$ for all k. The simplest iterative algorithm [1] to adjust the tap gains W_i^-, $-N_1 \le i \le 0$ is to change the ith tap by $-\Delta$ sgn $(\hat{e}_i)$, where Δ is the appropriately chosen constant step size, and $\hat{e}_i$ is an estimate of e_i, where $e_i = h_i$ for $-N_1 \le i \le -1$ and $e_0 = h_0 - 1$ (Fig. 7). An estimate $\hat{e}_i$ can be obtained, for example,

Table 1 Example of the precursor equalization.

Number of iterations	Tap coefficients		Unit response at equalizer output					
	w'_1	w_0	h_{-2}	h_{-1}	h_0	h_1	h_2	h_3
0	0	1	0	−0.1	1	−1	0.1	−0.05
1	0.05	1	0	−0.05	0.95	−1	0.1	−0.05
2	0.1	1.05	0	0	0.95	−1.05	0.1	−0.05
3	0.1	1.1	0	0	1	−1.10	0.1	−0.05

by cross-correlating the sequence $Z(D) - \hat{B}(D)$ with $D^{-i}\hat{B}(D)$ (the quantizer output $\hat{B}(D)$ shifted by i time units).

Table 1 shows how the precursor equalizer reaches the optimal tap setting. We chose the step size $\Delta = 0.05$ and for clear presentation neglected any values in the response smaller than Δ. The table shows that the precursor terms become less than Δ after three iterations. After its convergence, the unit response observed at the input of the main equalizer is given by the sequence $R(D)$ of Fig. 8. Now that the distortion of $R(D)$ is small, the same algorithm can be applied to adjust the main equalizer $W^+(D)$, and the equalizer operates exactly in the same manner as shown in Table 1.

In the discussion above we applied the precursor equalizer, $W^-(D)$, and then passed the resultant output into the main equalizer embedded in the inverse filter circuit. In actual implementation, however, adjustment of $W^+(D)$ and $W^-(D)$ can be performed simultaneously and independently.

5. Concluding remarks

In this presentation we have not elaborated on error detection and correction (or decoding) schemes. The scheme shown as an example in Fig. 7 is the conventional bit-by-bit detection with some error detection capability, which is discussed in great detail in our earlier results [8]. More sophisticated decoding algorithms such as ambiguity zone decoding [3,11] and maximum likelihood (or Viterbi) decoding [3,9,10,12,13] can be combined with the adaptive receiver discussed in the present paper. One could always expect a performance improvement by employing these sophisticated algorithms, since random noise will be the major source of error once all significant intersymbol interference components are removed by the adaptive equalizer.

In these decoding schemes, some additional delay is incurred in the decoding stage. Thus $B(D)$ of Fig. 7[or $A(D)$ of Fig. 4 in a system without precoding] would be provided by a tentative decoding output (e.g., the most likely "survivor" path in case of Viterbi's algorithm) rather than by the final decision output, which would often be available too late to be fed back to the main equalizer.

The scheme presented here can avoid error propagation in partial-response systems by adopting precoding techniques now well understood [1,5,6,8,11]. Recall, however, that precoding is possible only when g_0, g_1, $\cdots$, g_L are all integers with greatest common divisor equal to unity, and g_0 and m (the number of signal levels) are relatively prime. This should be taken into account in choosing the appropriate $G(D)$ for given m.

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