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Application of Partial-response Channel Coding to Magnetic Recording Systems

Abstract: A magnetic recording channel can be regarded as a "partial-response" channel because of its inherent differentiation in the readback process. The conventional NRZI method of recording is shown to be equivalent to the "precoding" of this particular partial-response channel, the purpose of which is to limit the propagation of error in the channel output. Using this new viewpoint, one can readily adopt an error detection scheme (developed for general partial-response channels) that takes full advantage of the inherent redundancy in the three-level channel output. The detection scheme is optimum in the sense that it detects all detectable errors with minimum delay.

The paper also describes a new high-density recording method, named the "Interleaved NRZI," which is obtained by molding an ordinary recording channel into a different type of partial-response channel, resulting in a potential increase in information density. Implementation of the corresponding optimum error detection scheme is also presented.

Finally, performance of these error detection schemes is evaluated in terms of probabilities of detecting single and double errors within a certain finite delay.

1. Introduction

Two of the most common methods of magnetic recording are the so-called NRZ (non-return-to-zero) and NRZI (non-return-to-zero-inverse) methods. In NRZ recording, one direction of magnetization corresponds to a ONE while the other corresponds to a ZERO in the data. In NRZI recording, a change in direction of magnetization corresponds to a ONE while no change corresponds to a ZERO in the data. Relationships among a sample of binary data, the writing current $i(t)$, the magnetization $m(t)$ of the recorded data and the readback voltage $e(t)$, for both NRZ and NRZI recording, are illustrated in Fig. 1. The relationship between the readback voltage $e(t)$ and the magnetization $m(t)$ can be described by the following expression[1]:

$$e(t) = h(t) * \frac{d}{dt} m(t), \tag{1}$$

where "*" means the convolution and $h(t)$ represents the magnetic-head field distribution characterized by the response due to a unit step function in $m(t)$. Note that h and m are written as functions of time t, under the assumption that the magnetic medium is moving at a constant speed relative to the reading head.

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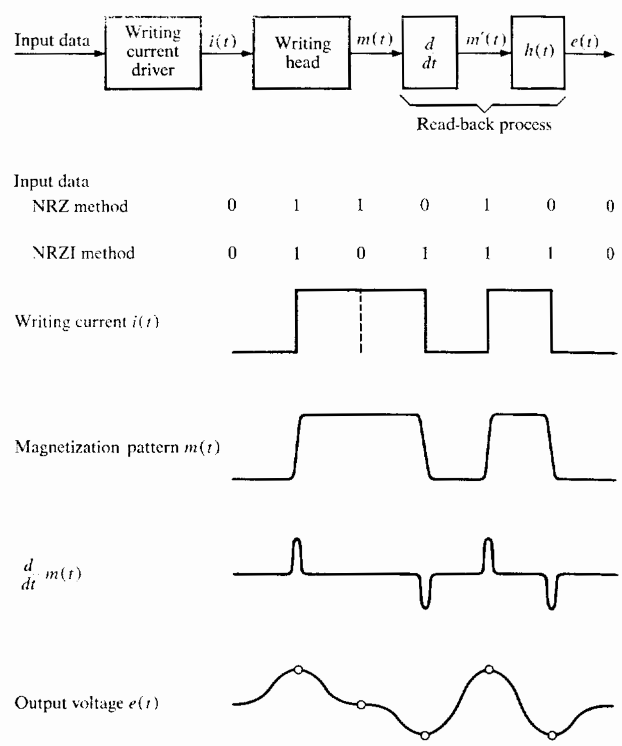
Figure 1 The NRZ and NRZI recording methods and the waveforms at various stages.



The main disadvantage of NRZ is that when a bit is misread in the readback process, the error propagates through the rest of the decoded message. It will be shown that because the readback process differentiates the signal, the readback channel can be considered a "partial-response" channel[2,3] that introduces a controlled amount of intersymbol interference between neighboring bits. A general precoding method can be applied to eliminate the problem of error propagation. It will be shown that this precoding method is equivalent to the NRZI method of recording.

As stated above, the conventional detection method of NRZI recording interprets the presence of a pulse in the readback voltage as a ONE and the absence of a pulse as a ZERO. This is often realized by passing the output voltage signal through a rectifier and then through a threshold detector. However, it is easy to see that the output voltage is basically a three-level signal with inherent redundancy that should be exploited for error control. The authors present an error detection scheme that makes use of these features of NRZI recording.

A new and somewhat more sophisticated method of high-density recording is also described in this paper. It is shown that an ordinary recording channel can be molded into a different type of partial-response channel. This can be done with or without channel shaping, and offers promise for an increase in information density with only a moderate sacrifice in signal power.

Information density of existing magnetic surface recording systems is limited mainly by the presence of intersymbol interference introduced in the readback process, an essentially linear process. The similarity between magnetic readback channels and data transmission channels is the basis for using modern communication techniques to improve the performance of magnetic recording systems. Results presented in this paper include several earlier pieces of work by the present authors on the treatment of partial-response channels and magnetic recording channels.

2. Partial-response channels

Partial-response channels, also called channels with correlative level coding, have been studied in recent years by several authors.[2-5] The methods of channel shaping, signalling and coding that they have developed are motivated by the need to better handle the problem of intersymbol interference and to utilize more efficiently the bandwidth of a given channel.

Partial-response channels have previously been studied by the present authors using the concept of linear digital filters that are defined either in the integer domain or in a residue class ring. In the following we give a brief mathematical description of the partial-response channel and outline some results in error detection that are applicable to magnetic recording of binary data. Detailed discussions of our error detection and correction techniques will be given in another paper.[6] A recent paper by Gunn and Lombardi[7] uses an approach to error detection in partial-response channels that is similar to ours.

A partial-response channel is characterized by a transfer function

$$G(D) \equiv \sum_{i=0}^{N} g_i D^i, \qquad (2)$$

where g_i are integers with a greatest common divisor equal to one, and D is a delay operator. Given an input sequence of integers $\{b_k\}$, the output sequence $\{c_k\}$ is determined by

$$C(D) = G(D)B(D), \qquad (3)$$

where

$$B(D) \equiv \sum_{k=0}^{\infty} b_k D^k \quad \text{and} \quad C(D) \equiv \sum_{k=0}^{\infty} c_k D^k.$$

Note that g_0 in $G(D)$ is taken as the signal value, while the g_i for $i \neq 0$ correspond to controlled interferences at the following digits. By allowing $g_i \neq 0$ for $i \neq 0$, the channel shaping becomes much easier in most cases when a higher information rate is desired. It can be seen that when $B(D)$ represents a binary sequence, $C(D)$ represents an m-level sequence, where m depends on coefficients in $G(D)$. For instance, the so-called duobinary system[2] corresponds to the case of $G(D) = 1 + D$, for which $m = 3$.

Now if the output $\hat{C}(D)$ is received without error, i.e., $\hat{C}(D) = C(D)$, then the input $B(D)$ can be recovered by passing $\hat{C}(D)$ through an inverse filter with a transfer function $1/G(D)$. An immediate problem of this system [Fig. 2(a)] is that if $\hat{C}(D)$ includes an error, the effect of this error tends to propagate in the decoded message $\hat{B}(D) = \hat{C}(D)/G(D)$. This can be seen from the fact that $1/G(D)$ can in general be expanded into an infinite power series in D. Propagation of errors can be avoided by the precoding technique.

The precoding technique can best be described in terms of a digital filtering process that is nonlinear in the ordinary sense, but is linear over the residue class ring modulo 2. We shall define a precoder by its transfer function, $[1/G(D)]_{\text{mod } 2}$. That is, with a binary input $A(D)$, the precoded message $B(D)$, which is also binary, is determined by

$$B(D) \equiv A(D)/G(D) \quad \text{mod } 2. \qquad (4)$$

The existence of this inverse filter requires the coefficient g_0 in $G(D)$ to be an odd integer. (In the general case, where the input is of m-level, g_0 and m must be relatively prime.) Comparing Eqs. (3) and (4), we see that

$$c_k \equiv a_k \quad \text{mod } 2, \text{ for all } k \geq 0. \qquad (5)$$

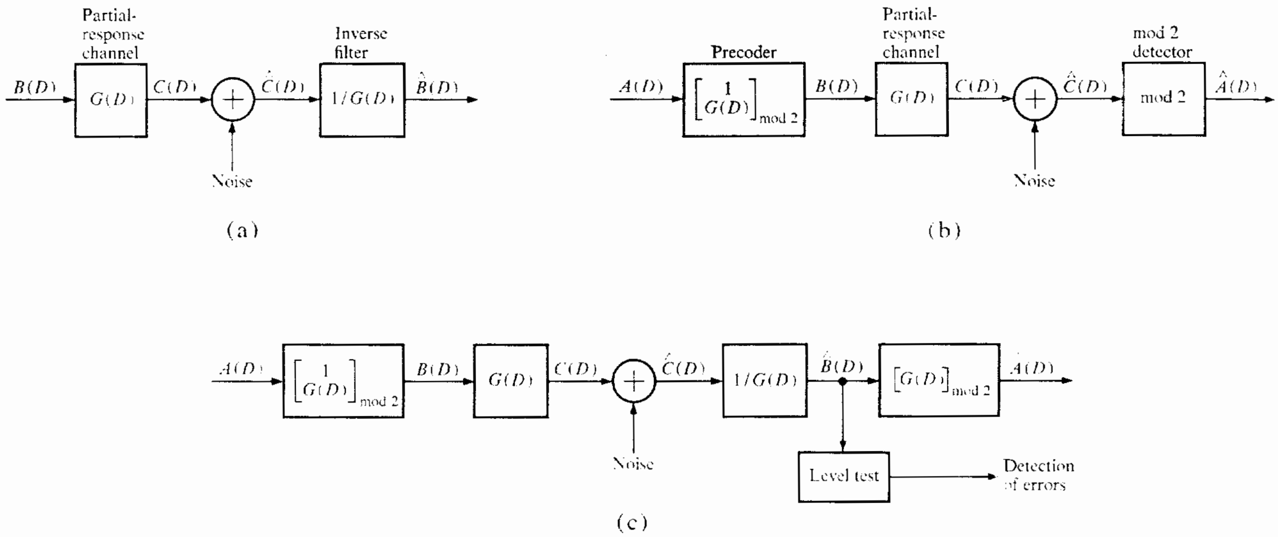


Figure 2 Partial-response system; (a) without precoding, (b) with precoding and (c) with error detection capability.

To recover $\hat{A}(D)$ given $\hat{C}(D)$, one merely performs a modulo 2 operation on $\{\hat{c}_k\}$. Propagation of errors in $\hat{A}(D)$ is thus eliminated.

In the system of Fig. 2(b), the detector simply performs a modulo 2 (or, equivalently, rectification) operation to recover the binary message $\hat{A}(D)$. Although extremely simple in structure, this detection method is not capable of taking advantage of the inherent redundancy in a multilevel output sequence $C(D)$ and will not detect any error. To remedy this weakness a new detection system is proposed [Fig. 2(c)]. In this modified receiving structure, the modulo 2 operator of Fig. 2(b) is replaced by the inverse filter $1/G(D)$ and the decoder $[G(D)]_{\text{mod } 2}$. It is not difficult to see that the inverse filter and the decoder together perform an equivalent modulo 2 operation, and so the objective of avoiding error propagation is still preserved.

If there are no errors in the channel output $\hat{C}(D)$, the inverse filter output $\hat{B}(D)$ is clearly the same as $B(D)$, the precoder output. When an error exists in $\hat{B}(D)$, it is said to be "detectable" if it cannot have been generated by any legitimate input $A(D)$ in the absence of errors. Detectable errors can always be detected in the receiving system of Fig. 2(c) simply by tracking the existence of any illegitimate coefficients in $\hat{B}(D)$. The validity and the optimality of such a detection scheme are guaranteed by the following theorem, the proof of which is given elsewhere.[6]

Theorem: Any detectable error in the channel output $\hat{C}(D)$ must result in an inverse filter output $\hat{B}(D)$ that contains a coefficient of level other than the allowable levels ZERO and ONE. Furthermore, the delay between the occurrence of an error and its detection is as small as possible.

The model of partial-response channels will next be used for digital magnetic recording systems.

3. The NRZI recording system with error detection capability

The relation between a binary sequence $\{b_k\}$ ($b_k = 1, 0$) and the writing current $i(t)$ in the NRZ recording system is given by

$$i(t) = \sum_{k=0}^{\infty} (2b_k - 1)u(t - kT), \tag{6}$$

where $u(t)$ is a rectangular pulse of duration T seconds:

$$u(t) = \begin{cases} 1, & 0 \le t < T \\ 0, & \text{elsewhere.} \end{cases} \tag{7}$$

Here, the amplitude in the current $i(t)$ is normalized by its saturation level, i.e., +ONE and −ONE represent two saturation levels. We may also assume that the magnetic medium is originally magnetized to one of these two levels, say, −ONE.

Although transitions between opposite saturation levels require some finite time, it is generally adequate to assume $m(t)$ to be approximately proportional to $i(t)$ in practical systems. The output voltage of the reading head, $e(t)$ of Eq. (1), can be shown in the absence of noise to be

$$e(t) = 2 \sum_{k=0}^{\infty} c_k h(t - kT), \tag{8}$$

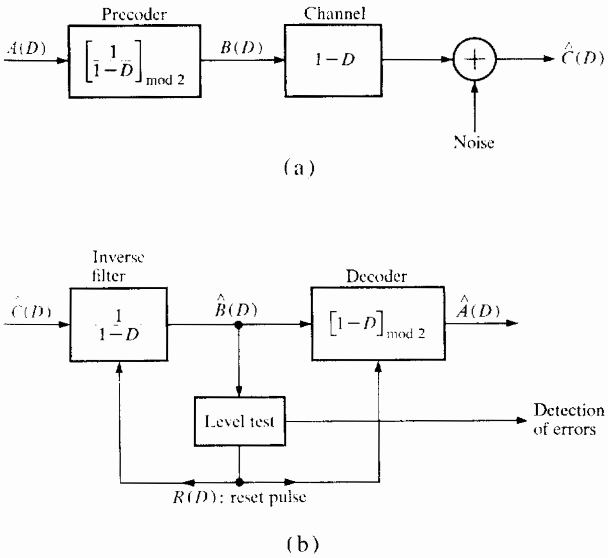


Figure 3 (a) A representation of the NRZI recording system as a partial-response system; (b) error detection and decoding method in the NRZI recording system.

where

$$c_k = \begin{cases} b_k - b_{k-1}, & k \geq 1 \\ b_0, & k = 0. \end{cases} \qquad (9)$$

If the response function $h(t)$ virtually satisfies the following condition at sampling instants,

$$h(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0, \end{cases} \qquad (10)$$

then the sampled value of the output voltage is

$$e(nT) = 2c_n, \qquad (11)$$

which is a three-level sequence, as is clear from Eq. (9). Thus, the recording channel can be regarded as a partial-response channel with a discrete transfer function

$$G_0(D) = 1 - D. \qquad (12)$$

From our earlier discussion on partial-response channels, the corresponding precoder here should perform a transformation that is "linear" in the binary sense, and has a transfer function $[1/(1 - D)]_{\text{mod }2}$. More specifically, the precoder input $\{a_k\}$ is related to its output $\{b_k\}$ by

$$b_k \equiv a_k + b_{k-1} \qquad \text{mod } 2$$

$$\equiv \sum_{i=0}^{k} a_i \qquad \text{mod } 2, \qquad (13)$$

with b_{-1} usually assumed to be zero.

It is easy to see that the precoder obtained above by viewing the magnetic channel as a partial-response channel performs exactly the function of an NRZI encoder [Fig. 3(a)]. That is, a symbol in the precoded binary sequence $\{b_k\}$ is always reversed from the preceding symbol when a ONE in the binary input sequence $\{a_k\}$ is to be recorded. This new observation, nevertheless, enables one to employ the general detection method developed for partial-response channels and to take full advantage of the inherent redundancy in the three-level channel output.

A possible structure of the error detection method based on the theorem in Section 2 is diagrammatically shown by Fig. 3(b). Here the rectifier or "mod 2 detector" in the conventional system is replaced by the combination of the inverse filter with transfer function $(1 - D)^{-1}$ and the decoder with transfer function $[1 - D]_{\text{mod }2}$.

It should be pointed out that, in this special case, the error criterion is equivalent to the observation that voltage pulses read from a saturation recording system must alternate in polarity. However, it is reassuring to know that *all* detectable errors can be detected this way. The special form used to implement the error detection logic may also be of some practical interest.

4. A new high-density recording scheme

The function $h(t)$ of Eq. (1) is the output voltage response to an impulse in $d[m(t)]/dt$; i.e., to a step change in the direction of recording medium saturation. It is known that a Gaussian characteristic, $h(t) = a \exp(-bt^2)$, can generally provide a good fit to experimentally obtained characteristic voltage pulses.[1] It is seen from Fig. 4(a) that although the pulse shape $h(t)$ has no overshoot, it is not suitable for high-density recording, since a bit interval T of a fairly large value must be chosen to avoid excessive intersymbol interference.

The pulse in Fig. 4(c) is the same as that of Fig. 4(b). However, the sampling rate is increased by 50 percent; i.e., sampling is done at every T' second, where $T' = \frac{2}{3}T$. If we define the new time axis t' by

$$t' = t + \tfrac{1}{2}T' \qquad (14)$$

and a function $f(t')$ by

$$f(t') = h(t + \tfrac{1}{2}T'), \qquad (15)$$

then, as can be seen from Fig. 4(c)

$$f(nT')\begin{cases} = 1, & n = 0, 1 \\ \approx 0, & n \neq 0, 1, \end{cases} \qquad (16)$$

where the values at $n = 0, 1$ are normalized. Hence, the transfer function is characterized by

$$F(D) = \sum_{n=-\infty}^{\infty} f(nT')D^n = 1 + D. \qquad (17)$$

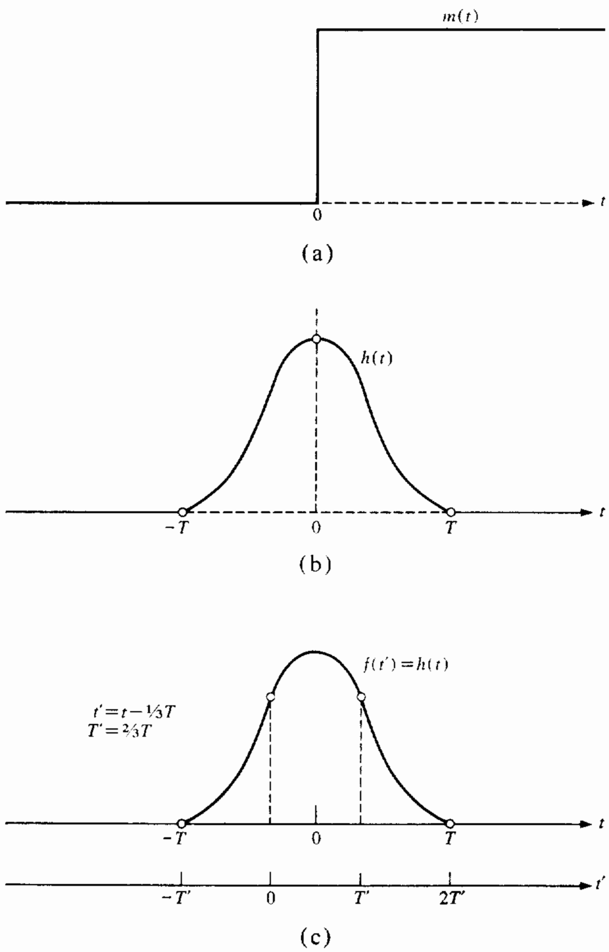
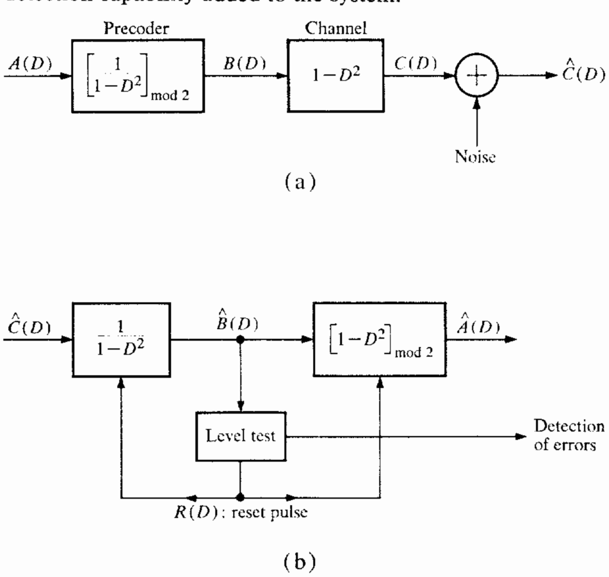


Figure 4 The principle of a higher-density recording system.

Figure 5 (a) Interleaved NRZI recording method; (b) error detection capability added to the system.



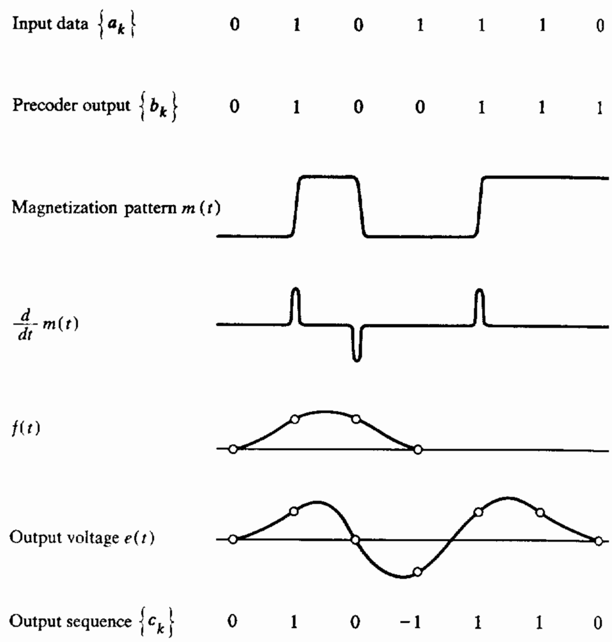


Figure 6 Waveforms at various stages in the Interleaved NRZI recording system.

Then the transfer function of the whole system is, from Eqs. (12) and (17),

$$G_1(D) = G_0(D)\,F(D) = 1 - D^2. \tag{18}$$

The corresponding precoder has a transfer function $[1/(1 - D^2)]_{\text{mod } 2}$. The input sequence $\{a_k\}$ and output sequence $\{b_k\}$ are related by

$$b_k = a_k + b_{k-2} \quad \text{mod } 2. \tag{19}$$

We shall call this new system the "Interleaved NRZI" recording system. The block diagram of the system is shown in Fig. 5. Fig. 6 shows the actual waveforms observed at various stages in the Interleaved NRZI system.

The Interleaved NRZI system described above requires the channel response to satisfy Eq. (16). In practice, however, such desired channel response can not always be maintained (especially if the sampling rate is pushed higher), and, as a result, system performance may deteriorate. A common remedy is the introduction of "shaping" or "equalization" in the frequency characteristic of the channel. A possible frequency characteristic $H(\omega)$ [the Fourier transform of the channel response function $h(t)$] assumes a cosine shape in the magnitude, as shown in Fig. 7, while possessing a linear phase characteristic. Equalization or channel shaping can also be realized in the time domain. Automatic or adaptive equalization developed in data transmission systems[8,9] can thus be applied to the magnetic recording channel. However, this subject is beyond the scope of the present paper.

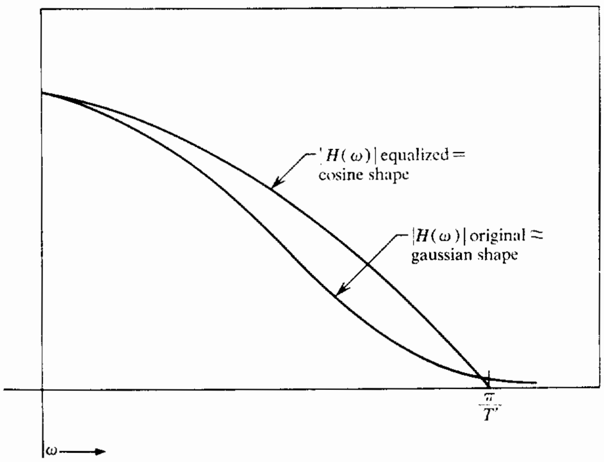
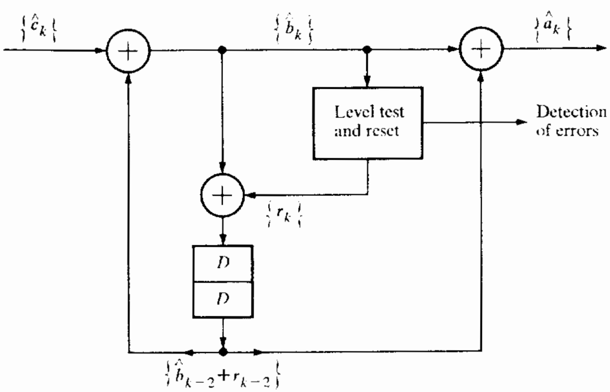
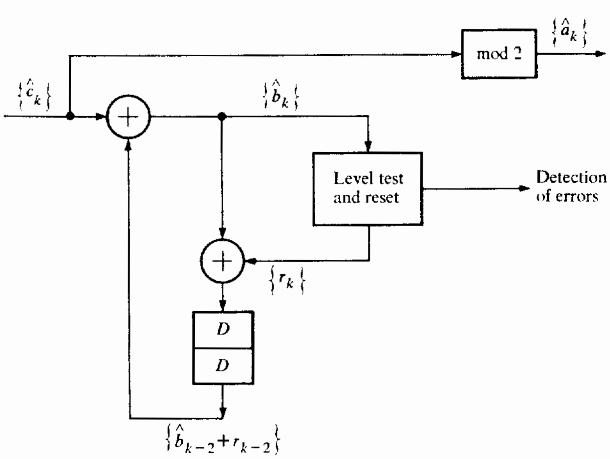


Figure 7 Frequency characteristic of the channel response function.

Figure 8 (a) An error detection and decoding circuit for the Interleaved NRZI system; (b) an alternate method.



(a)



(b)

Table 1 Example of operation of Interleaved NRZI system.

k	0	1	2	3	4	5	6	7	8
a_k	0	1	1	0	0	0	1	1	1
$b_k \equiv b_{k-2} + a_k \pmod 2$	0	1	1	1	1	1	0	0	1
$c_k = b_k - b_{k-2}$	0	1	1	0	0	0	−1	−1	1
$\hat{c}_k$	0	1	0	0	0	0	−1	−1	1
$\hat{b}_k = \hat{c}_k + (\hat{b}_{k-2} + r_{k-2})$	0	1	0	1	0	1	−1	0	1
r_k	0	0	0	0	0	0	1	0	0
$\hat{a}_k = \hat{b}_k + (\hat{b}_{k-2} + r_{k-2}) \equiv \hat{c}_k \pmod 2$	0	1	0	0	0	0	1	1	1

Fig. 8(a) shows a possible implementation of the error detection and decoding circuit for the Interleaved NRZI system. Note that a two-stage shift register is shared by the inverse filter and the decoder. Figure 8(b) shows an alternative implementation in which the output sequence $\{\hat{a}_k\}$ is directly obtained from $\{\hat{c}_k\}$. The generation rule of the reset pulse $\{r_k\}$ is as follows:

$$r_k = \begin{cases} -(\hat{b} - 1), & \hat{b} > 1 \\ 0, & \hat{b} = 1 \text{ or } 0 \\ -\hat{b}, & \hat{b} < 0. \end{cases} \tag{20}$$

The operation of this proposed system will be illustrated by the following example.

• *Example 1:* $G(D) = 1 - D^2$

Let us assume that the information sequence $\{a_k\}$ is as given in Table 1. Then, the precoded sequence $\{b_k\}$ is obtained by Eq. (19); the channel output sequence $\{\hat{c}_k\}$ is calculated by assuming the transfer function of Eq. (18). In Table 1, $\hat{c}_2$ is in error and the corresponding error in $\{\hat{b}_k\}$ propagates in every other bit as marked by the dotted line and results in an illegitimate level $\hat{b}_6 = -1$ with a four-digit delay. A reset pulse r_6 is generated to prepare the system for future errors.

The main features of the Interleaved NRZI recording system are summarized as follows.

1) Without any channel shaping the recording density can be increased by properly changing the sampling rate and time in the conventional NRZ or NRZI recording system.

2) Density can be further increased by equalizing the channel. Since the frequency characteristic of a recording channel is fairly close to the desired cosine shape, this type of equalization becomes somewhat easier. As a result, the signalling rate can be substantially increased without resulting in excessive sensitivity problem.

3) Precoding can be used to prevent propagation of errors. This can be easily implemented with simple digital circuitry.

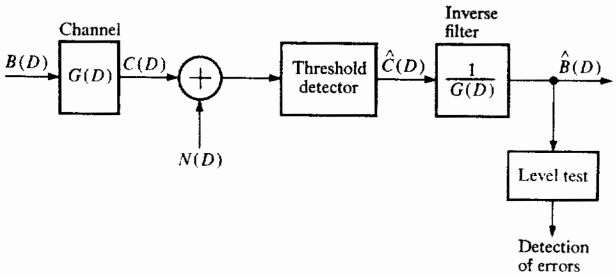


Figure 9 Threshold detection.

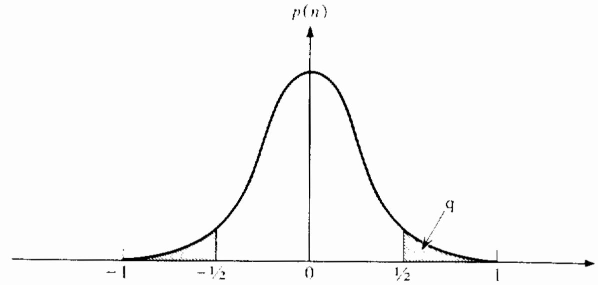


Figure 10 Probability density function of the additive noise $\{n_k\}$.

4) Error detection and correction methods developed for a general partial-response system can be used here.

Furthermore there are other desirable properties similar to those in the NRZI method:

5) In parallel recording on a group of tracks, an odd parity count per data frame guarantees a signal for readback timing in every two-bit period.
6) Because signal polarity is not a factor in interpreting the readback voltage, reading is simplified when bidirectional surface motion is used.

It is worth mentioning here that the approach discussed in the present section can be further generalized; i.e., by the proper choice of the recording density (bit rate) and by equalization of the channel frequency characteristic, a wide class of partial-response transfer functions of the form

$$G(D) = G_0(D)\,P(D) \tag{21}$$

can be obtained, where $G_0(D) = 1 - D$ [see Eq. (16)] and $P(D)$ is any polynomial of D with an odd constant term. A possible choice, for instance, could be

$$P(D) = 1 + 2D + D^2. \tag{22}$$

The potential increase in recording density in this case is even higher than that of the Interleaved NRZI technique. However, one must accept any degradation in performance due to the increased number of channel output levels. Other practical considerations, including the ease of proper equalization, are also open to further investigation.

5. Performance of error detection algorithms in the NRZI and the Interleaved NRZI recording systems

Thus far we have not specified channel error statistics, nor have we characterized the "noise" in our system to have any specific properties. Furthermore, we have not indicated how the output sequence $\{\hat{c}_k\}$ is derived from the read-head output analog waveform except that the output $\{\hat{c}_k\}$ should be equal to $\{c_k\}$ under noiseless conditions.

Let us assume now that the error source is described by an additive noise sequence $\{n_k\}$ that is independent of the information sequence $\{c_k\}$. We assume that the threshold detection method will be used as shown in Fig. 9, i.e.,

$$\hat{c}_k = \begin{cases} 1, & c_k + n_k \geq \frac{1}{2} \\ 0, & |c_k + n_k| < \frac{1}{2} \\ -1, & c_k + n_k \leq -\frac{1}{2}, \end{cases} \tag{23}$$

in which the received signal spacing is normalized to unity and, hence, the threshold levels are given by $\pm\frac{1}{2}$. We further assume that the noise distribution density function is symmetrical about $n_k = 0$ (Fig. 10). Thus,

$$\Pr\{n_k > \tfrac{1}{2}\} = \Pr\{n_k < \tfrac{1}{2}\} \equiv q. \tag{24}$$

If the sequence $\{a_k\}$ is an independent sequence in which ONE and ZERO occur with equal probability, so is the precoded sequence $\{b_k\}$. The output sequence $\{c_k\}$ will take levels +ONE, ZERO and −ONE with probabilities $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$, respectively. The probability that a detected bit $\{\hat{c}_k\}$ is in error is

$$\begin{aligned} p_e = {} & \tfrac{1}{4}\Pr\{\hat{c}_k = 0 \mid c_k = 1\} \\ & + \tfrac{1}{4}\Pr\{\hat{c}_k = 0 \mid c_k = -1\} \\ & + \tfrac{1}{2}\Pr\{\hat{c}_k = 1 \mid c_k = 0\} \\ & + \tfrac{1}{2}\Pr\{\hat{c}_k = -1 \mid c_k = 0\} = \tfrac{3}{2}q. \end{aligned} \tag{25}$$

The probability $p_d(0)$ that an error in the NRZI system is immediately detectable is given by

$$\begin{aligned} p_d(0) & = \Pr\{\hat{b}_k = 2\} + \Pr\{\hat{b}_k = -1\} \\ & = \tfrac{1}{4}\Pr\{\hat{c}_k = 1 \mid b_k = b_{k-1} = 1\} \\ & \quad + \tfrac{1}{4}\Pr\{\hat{c}_k = -1 \mid b_k = b_{k-1} = 0\} \\ & = \tfrac{1}{2}q = \tfrac{1}{3}p_e \end{aligned} \tag{26}$$

Extending the argument, we say that a single error can be detected after a delay of exactly L digits with probability

$$p_d(L) = \{p_e - p_d(0)\}(\tfrac{1}{2})^L = \tfrac{1}{3}P_e(\tfrac{1}{2})^{L-1}, \quad L \geq 1. \tag{27}$$

The conditional probability that a single error can be detected within a delay L is given, from Eqs. (25) and (26), by

$$p_d(L \mid \text{single error}) = \frac{1}{p_e} \sum_{l=0}^{L} p_d(l) = 1 - \tfrac{1}{3}(\tfrac{1}{2})^{L-1}, \qquad L \geq 0. \qquad (28)$$

In the above argument we assume that all errors are "single" errors; i.e., that each error is separated from the next one by more than L digits. What if another error occurs within L digits after the first one? If the second error has the same polarity as the first one, corresponding error patterns in the sequence $\{\hat{b}_k\}$ will add. Thus $\{\hat{b}_k\}$ always exceeds legitimate levels no later than the occurrence of the second error. Therefore, these errors can be detected with probability one. If two errors have opposite polarities, however, error patterns cancel each other after the occurrence of the second error; thus, these errors remain undetected unless the first error is detected before the occurrence of the second error. This probability is given by

$$1 - \tfrac{1}{3}(\tfrac{1}{2})^{s-2}, \quad L \geq s \geq 1, \qquad (29)$$

where s is the separation of two errors. Hence, the probability of double errors being detected within an L-digit delay is given by

$$p_d(L \mid \text{double errors}) = \tfrac{1}{2} + \tfrac{1}{2}(1/L) \sum_{s=1}^{L} p_d(s-1 \mid \text{single error}) = 1 - \tfrac{1}{3}(1/L)[1 - (\tfrac{1}{2})^L]. \qquad (30)$$

The performance of the Interleaved NRZI system can be analyzed in a similar manner: If, in the Interleaved NRZI system, one decomposes $\{a_k\}$ into two subsequences, each subsequence can be treated as if it were an input to the NRZI system. Hence, the probability that a single error can be detected within a delay L (assumed to be even) is given, from Eq. (28), by

$$p_d(\tfrac{1}{2}L \mid \text{single error}) = 1 - \tfrac{1}{3}(\tfrac{1}{2})^{L/2-1}, \quad L \geq 0. \qquad (31)$$

Similarly, the double-error detection probability is given by

$$\tfrac{1}{2}p_d(\tfrac{1}{2}L \mid \text{double errors}) + \tfrac{1}{2}[1 - \{1 - p_d(\tfrac{1}{2}L \mid \text{single error})\}^2] = 1 - \tfrac{1}{9}(\tfrac{1}{2})^{L-1} - \tfrac{2}{3}(1/L)\{1 - (\tfrac{1}{2})^{L/2}\}. \qquad (32)$$

6. Concluding remarks

We have shown in this paper the following results:

1) A magnetic recording channel can be treated as a partial-response channel that possesses an inherent factor $(1 - D)$ in its discrete transfer function.
2) An error detection method applicable to a partial-response channel is described and is applied to the NRZI system.
3) A novel high-density recording method, called the "Interleaved NRZI" recording system, has been devised and its properties are discussed.
4) The performance of the proposed error detection scheme applied to the NRZI and Interleaved NRZI systems is analyzed.

It should be noted that although the potential improvement in using the proposed "Interleaved NRZI" method is demonstrated, many practical problems in implementing such a system (e.g., detection and synchronization techniques, etc.) are still open for further investigation.

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Received August 21, 1969