THE MAXIMUM LIKELIHOOD DECODING OF CORRELATIVE LEVEL CODED SEQUENCES

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Summary. A correlative level encoder is treated as a linear finite state machine and an application of the maximum likelihood decoding algorithm is discussed. Asymptotic expressions for the probability of decoding error are obtained for a class of correlative level coding systems, and the results are confirmed by computer simulations. It is shown that a substantial performance gain is attainable by this probabilistic decoding method.

Introduction

A technique in digital data communication developed in recent years is the so-called correlative level coding (Lender) or the partial-response channel signaling (Kretzmer). It has been widely accepted that although such a signaling method possesses the property of being insensitive to channel imperfections, the increase in the number of signal levels results in loss of noise margins.

Recently an analogy between correlative level coding and convolutional coding has been pointed out by Kobayashi, Tang, and Forney. A correlative level encoder can be viewed as a simple type of linear finite state machine defined over the real number field as opposed to a Galois field over which a convolutional encoder is defined. The present paper will show that the maximum likelihood decoding algorithm devised by Viterbi in decoding convolutional codes is applicable to our problem. Both analytical and experimental results of this probabilistic decoding scheme will be presented. The performance of the maximum likelihood decoding (MLD) is much superior to any other method reported thus far. Asymptotic expressions for the decoding error probability are derived. Several other important problems associated with the MLD method are discussed: the effect of precoding on the decoding error rate and error patterns, the number of quantization levels required, and the problem of decoder buffer overflows.

The Maximum Likelihood Decoding of Correlative Level Coded Sequences

In the present paper we limit ourselves to the correlative level coding system $G(D) = 1 - D$ (which corresponds to a digital magnetic recording channel). The so-called modified duobinary or the partial response class IV correspond to the transfer function $G(D) = 1 - D^2$, which is merely an interleaved form of $1 - D$. The duobinary signaling $1 + D$ holds a dual relationship with $1 - D$. Thus the basic decoder structure and the performance are common to the class

$$G(D) = 1 \pm D^K, \quad K = 1, 2, \cdots .$$

Let $\{a_k\}$ be the information sequence which takes on integer levels $\{0, 1, \cdots, m-1\}$. We define the state of the encoder by

$$s_k = a_k \quad \text{(without precoding)} \qquad (1)$$

or

$$s_k = s_{k-1} + a_k \mod m \quad \text{(with precoding)} \qquad (2)$$

If we assume that $\{a_k\}$ takes on $0, 1, \cdots, m-1$ with equal probability and independently and that the additive noise of the channel is Gaussian and is independent at sampled instants, the maximum likelihood decoding algorithm is simply a repetitive use of the following recursive formula, which determines a unique survivor for each of m different state nodes at time k. (See Ref. 8 for the derivation of Eqs. (3)&(4)

$$\mu_k(j) = \max_i \{ \mu_{k-1}(i) + (j-i)\, y_k - (j-i)^2 \tfrac{A}{2} \} \qquad (3$$

$$j = 0, 1, \cdots, m-1 \text{ and } k \geq 1.$$

and

$$\mu_0(j) = \begin{cases} 0 & \text{for } j = s_0 \\ -\infty & \text{for } j \neq s_0 \end{cases} \qquad (4$$

where $\mu_k(j)$ is the metric of the survivor which ends at state j at time k, and y_k is the decoder input at time k. Constant A represents the signal spacing in the channel.

Performances of the Maximum Likelihood Decoder

In the present section we present analytical results on the performance of the MLD algorithm and then computer simulation results will be reported to confirm the analytical results.

Let us assume a high SNR (signal-to-noise ratio) condition. Then we consider, as possible adversary paths (paths competing against the correct path) only those which are "closest" to the correct path. Since the slope of the trellis correspond to the signal level in the channel, adversaries are those which stay closes to, and in parallel with, the correct path. After some manipulation, we obtain the following symptotic expressions for the decoding error rate when the MLD algorithm of Eqs. (3) and (4) is adopted:

$$P_{MLD} = 4(m-1)^2\, Q\left(\sqrt{\frac{3R}{m^2-1}}\right) \quad \text{(without precoding)} \qquad (5$$

and

$$P'_{MLD} = 4(m-1)\, Q\left(\sqrt{\frac{3R}{m^2-1}}\right) \quad \text{(with precoding)} \qquad (6$$

where R is the channel SNR : $R = \frac{(m^2-1)}{6} \frac{A^2}{s^2}$. Except fo m = 2 (i.e. binary inputs), $P'_{MLD} < P_{MLD}$. Thus precoding is beneficial not only in the conventional bit detection but also in the MLD method. In the conventional bit detection method the error rate is given by

$$P_{BIT} = 2\left(1 - \frac{1}{m^2}\right) Q\left(\sqrt{\frac{3R}{2(m^2-1)}}\right) \qquad ($$

It will be interesting to compare these results with a m-level PAM system without correlative level coding. The expression for the error rate is

$$P_m = 2\left(1 - \frac{1}{m}\right) Q\left(\sqrt{\frac{3R}{m^2-1}}\right) \qquad ($$

In case of m = 2, for example, P'_{MLD} is only four times of P_m, thus the MLD method allows a PAM system t adopt a correlative level coding technique to attain some desired spectral shaping with a very little penalty in its performance. In other words the loss in noise margin can be almost completely recovered by the MLD method.

Some Practical Considerations

Thus far we have tacitly assumed that the receiver input y_k is quantized into infinitely many levels. In an actual implementation which is presumably in a digital form, the channel output y_k must be quantized into a finite number of levels. If a uniform quantization is performed, the metric computation can be done in the integer format. The performance of the MLD for a system with $G(D) = 1-D$ and $m = 2$, is obtained when the quantization spacing is A/N where $N = 4, 8, 16,$ and 32. According to this result we may conclude that $N = 16$ achieves almost the same performance (less than 0.1 dB loss) as the infinite level quantization.

The second problem which would arise in implementation will be the memory size required in the decoder. The decoder can store only a finite history of m surviving paths. Let t and t' ($t' \geq t$) be times at which m survivors branch out of a common node. Then for a binary input system (i.e. $m = 2$) the distribution of separation $s = t' - t$ is given by

$$P(s) = 2^{-s} \left[\frac{2}{s} - \frac{1}{s+1}\right] \cong \frac{1}{s} 2^{-s} \qquad (9)$$

References

1. A. Lender, "Correlative Level Coding for Binary Data Transmission", IEEE Spectrum, Feb., 1966.

2. E. R. Kretzmer, "Generalization of a Technique for Binary Data Transmission", IEEE Trans. on Communication Technology, Feb., 1966.

3. H. Kobayashi and D. T. Tang, "On Decoding and Error Control of Correlative Level Coding", 1970 International Symposium on Information Theory, June, 1970.

4. G. D. Forney, Jr., "Error Correction for Partial Response Modems", 1970 International Symposium on Information Theory, June, 1970.

5. A. J. Viterbi, "Error Bounds for Convolutional Codes and Asymptotically Optimum Decoding Algorithm", IEEE Trans. on Information Theory, April, 1967.

6. A. J. Viterbi, "Convolutional Codes, Linear Finite-State Machine and Maximum Likelihood Decoding", to be published.

7. H. Kobayashi, "Application of Probabilistic Decoding to Digital Magnetic Recording Systems", IBM Research Report RC-2986, August, 1970. Also to appear in IBM Journal of Res. and Dev., Jan., 1971.

8. H. Kobayashi, "Correlative Level Coding and The Maximum Likelihood Decoding", IBM Research Report RC-2999, August, 1970.

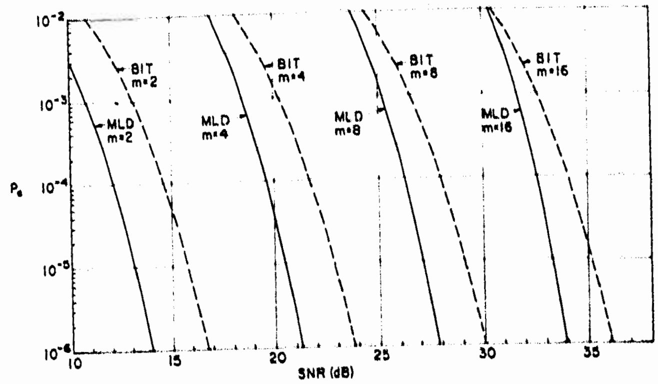

Figure 1. Comparison of the MLD Method and the Bit Detection Method

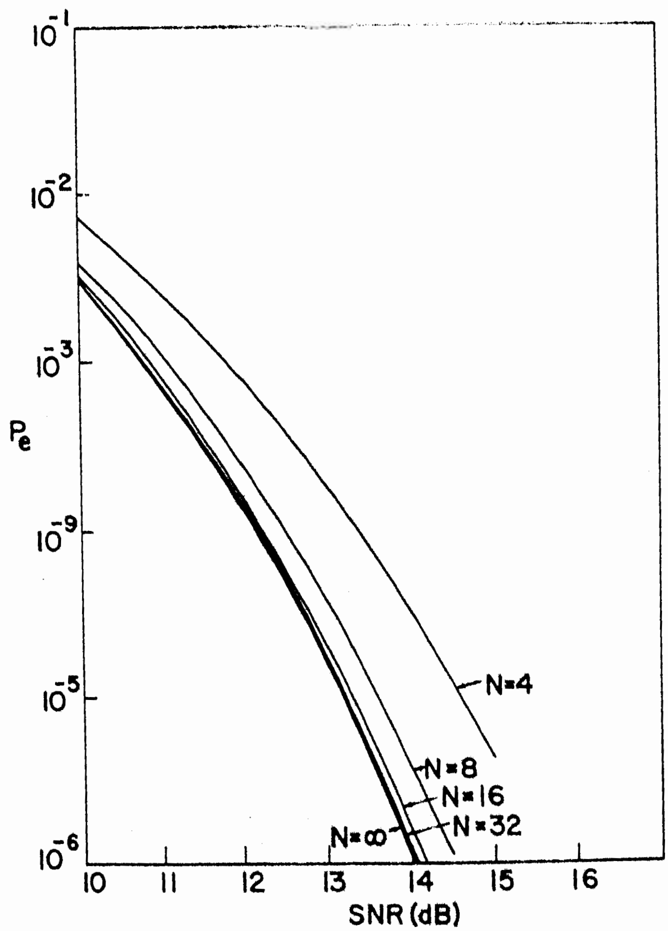

Figure 2. Effect of Finite Quantization Levels

($m=2$: quantization spacing $= \frac{A}{N}$)