that the theory of permutation decoding can be extended to arbitrary linear codes using arbitrary permutations with the only demand, that a permuted codeword be a codevector of an equivalent code.

Because of a relatively low amount of hardware, code invariant permutations seem to be of most practical interest, though being not generally capable to move all errors into the redundant positions. It will be shown secondly, that in case of cyclic codes which cannot be decoded successfully by the cyclic and quadratic permutations a combination with an error-pattern superposition in the information symbols will lead to the desired result. The idea is to move by means of a suitable permutation a maximum of errors into the redundant positions and to find the rest of errors in the information symbols by a suitable error pattern superposition. A criterion for successful superposition will be given.

9.3 A Decision-Directed Decoder for a Binary Symmetric Channel with Unknown Crossover Probability, *D. D. Falconer* (Bell Telephone Laboratories, Holmdel, New Jersey). We consider an example of a decision-directed decoder for a channel with an initially unknown parameter. A stream of information symbols is coded into a sequence of N-bit block code words and transmitted over a memoryless binary symmetric channel with an unknown crossover probability p. Before decoding a newly received block, the decoder computes a new estimate of p which is based on all past decoding decisions. It uses this running estimate to compute the a posteriori probabilities of all code words. The decoder's output is then a list of "hypothesized code words" which are all those (if any) whose computed a posteriori probabilities exceed a fixed threshold. This somewhat artificial "threshold decoding" operation is similar in principle to use of the path metric criterion in the Fano sequential decoding algorithm for convolutional codes.

The decoder's current estimate of p is taken to be the ratio of the cumulative error count to the total number of binary digits transmitted; the cumulative error count is the sum of the Hamming distances between all hypothesized words and the corresponding received blocks up to the current time. This estimator of p can be written in recursive form and is in fact a form of the Robbins-Monro stochastic approximation procedure. The estimate is shown by a random coding argument to be asymptotically (for large N) unbiased provided it never falls below a certain critical value p_c. This critical value is a function of p and of the decision threshold; it approaches p from below as the code rate (which controls the decision threshold) approaches the channel capacity. If the estimate ever drops below p_c it tends to continue drifting toward zero unless corrective action is taken.

The probability that the estimate of p ever drops below p_c is shown to be bounded by an expression that approaches zero exponentially with block length. The probability that either the correct code word does not appear on the decoder's list or that any incorrect code word does appear is shown to be asymptotically upper-bounded by the same expression that arises in the case where p is known.

9.4 On Decoding and Error Control for a Correlative Level Coding System, *H. Kobayashi* (University of California, Los Angeles, California) and *D. T. Tang* (IBM Research Center, Yorktown Heights, New York). A technique in data communication developed in recent years is the so-called partial-response signalling or the correlative level coding in which a controlled amount of intersymbol interference is utilized to improve the information rate. Such a system is characterized by the transfer function $G(D) = \Sigma_{i=0}^{N} g_i D^i$, where D^i means delay of i time units and $\{g_i\}$ is a set of integers. Recently it has been pointed out by the present authors that a digital magnetic recording channel can be regarded also as a partial-response channel with $G(D) = 1 - D$ due to its inherent differentiation in the read-back process. With equalization, the equivalent of $G(D) = 1 - D^2$ can be obtained for high-density recording. This paper discusses certain important problems of such coding systems from both algebraic and probabilistic approaches.

First, an algebraic treatment is given to the precoding, decoding and error control. Precoding which eliminates error propagation is discussed in terms of a linear discrete filter defined over a residue class ring. An error detection scheme which makes full use of the inherent redundancy in the correlative level coded sequence is then discussed. In this scheme a "modulo m" detector in the conventional receiver (where m is the number of information sequence levels) is replaced by the inverse filter and the decoder. If any detectable error exists in the received sequence, it can be always detected simply by tracking the existence of any illegitimate level in the inverse filter output sequence. The validity and the optimality of such an error detection scheme is proved.

This algebraic algorithm is extended to an error correction scheme in which the receiver makes a soft decision. Here, the number of quantization levels in the receiver is increased from M to $2M - 1$, including $(M - 1)$ ambiguity levels. (Here $M = (m - 1) \Sigma_{i=0}^{N} | g_i | + 1$ is the number of signal levels at the correlative level coder output.) It is shown that a wide class of errors can be corrected without resorting to retransmission.

Finally, another type of decoding is discussed; namely, the maximum likelihood decoding algorithm based on the linear finite-state machine model of the correlative level encoder. Techniques developed by Viterbi in decoding convolutional codes have been applied to our problem. This probabilistic decoding method has an advantage that the decoder structure is subject to no change whether the receiver makes a soft decision or a hard decision. Its performance is compared with that of the algebraic scheme in terms of information bit error probability.

9.5 Encoding and Decoding for Binary Input, M-ary Output Channels, *E. J. Weldon, Jr.* (University of Hawaii, Honolulu, Hawaii). In many communication systems the demodulator must make a "hard" binary decision after examining the received waveform. This hard decision causes a loss in channel capacity and, more importantly, a reduction in the error exponent at all rates below capacity. However, in systems using coding to improve reliability, decoding is considerably simpler if the decoder processes only binary digits. In practical situations this can more than compensate for the increased probability of error.

This talk presents a technique for decoding block codes in situations where the demodulator quantizes the received signal space into $M > 2$ regions. The method, referred to as Weighted Erasure Decoding, is in principle applicable to any block code for which a binary decoding procedure is known.

In section 1 of this talk, Weighted Erasure Decoding is introduced. In section 2 practical methods of implementing this decoding procedure are described. In section 3 we examine the performance of a specific code used on the additive white Gaussian noise (AWGN) channel and decoded with Weighted Erasure Decoding for various values of M. It is shown, as expected, that even small values of M yield substantial improvements over strictly binary decoding.

It is interesting to observe that all three of the practical decoding procedures for convolutional codes—sequential decoding, threshold decoding and Viterbi decoding—are readily adapted to M-ary output channels. On the other hand, Weighted Erasure Decoding is the first broadly applicable procedure for decoding block codes on M-ary output channels.

9.6 Analog Coding, *D. D. McRae, C. J. Palermo,* and *M. G. Pelchat* (Radiation Inc., Melbourne, Florida). Many modulation techniques such as angle modulation and PCM have the capability of achieving a signal-to-noise improvement over that available from single-sideband when the input signal-to-noise ratio is sufficiently high and bandspreading (relative to single-band) exists. The magnitude of the improvement, for these techniques, depends upon bandspreading. Shannon's rate distortion bounds, however, argue the existence of techniques for which the improvement for a given bandspreading also increases with input signal-to-noise ratio.

This paper describes a modulation technique which is both simple to implement and provides performance remarkably close to Shannon's bounds for bandspreadings over single sideband in the order of two or three to one. Curves of the performance of this system will be presented for the case where a simple receiver is used and for the case where a minimum-mean-square-error receiver is used.

The technique is a hybrid one in that it combines continuous and discrete modulation. The performance advantages which are provided can best be understood by extending the notions of "twisted modulation" as presented by Wozencraft and Jacobs. It is felt that this basic approach should yield considerable insight into the overall problem of signal design when a cost criteria other than minimum-error-probability is employed.

9.7 Decoding with Channel Measurement Information, *D. Chase* (General Atronics Corporation, Philadelphia, Pennsylvania). Block Coding techniques have been developed which enable one to correct all patterns of errors in a received block of binary digits provided that the number of errors in the block does not exceed some number. The maximum number of errors that can be corrected is generally less than one-half of the code's minimum Hamming distance.

In this paper a decoding algorithm which utilizes channel measurement information, in addition to the conventional use of the algebraic properties of the code, is presented. The maximum number of errors that can, with high probability, be corrected is equal to one less than the code's minimum Hamming distance. This two-fold