A Comparative Study of Pulse Combining Schemes for Impulse Radio UWB Systems[1]

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***Abstract*— Impulse radio systems transmit a number of ultra-wideband pulses for each information bit. In a multiuser scenario, the pulses from a given user may have different signal-to-interference-plus-noise ratio (SIR) and therefore a receiver that combines these signals with equal weight assignment is suboptimal. In this paper, a multiuser impulse radio system over an additive white Gaussian noise channel is considered and the performances of three types of receivers that employ different pulse combining schemes are investigated. Namely, a conventional matched filter receiver, a blinking receiver and a minimum mean square error (MMSE) receiver are considered. Assuming a large number of pulses per information bit, approximate expressions of bit error probability for the conventional matched filter and blinking receiver are presented. Then, conditions under which one receiver has a lower probability of error than the other are obtained. Also an MMSE receiver is proposed, which optimally combines the pulses but has higher complexity compared to the other types of receivers. The simulation studies are employed to compare the performances of the receivers and to verify the analytical expressions.**

***Index Terms*— Ultra-wideband (UWB), impulse radio (IR), multiple access interference (MAI), minimum mean square error (MMSE) combining.**

I. Introduction

Since the US Federal Communications Commission (FCC) approved the limited use of ultra-wideband (UWB) technology [1], communications systems that employ UWB signals have drawn considerable attention. A UWB signal is defined as one that possesses a bandwidth larger than 500MHz and can coexist with with incumbent systems in the same frequency range due to its large spreading factor and low power spectral density. UWB technology holds great promise for a variety of applications such as short-range high-speed data transmission and precise location estimation.

Commonly, impulse radio (IR) systems, which transmit very short pulses with a low duty cycle, are employed to implement UWB systems ([2]-[4]). In an IR system, a train of UWB pulses is sent and information is usually conveyed by the positions or the polarities of the pulses. In order to prevent catastrophic collisions among different users and thus provide robustness against multiple access interference, each information symbol is represented by a sequence of pulses; the positions of the pulses within that sequence are determined by a pseudo-random time-hopping (TH) sequence specific to each user [2]. The number N_f of pulses representing one information symbol can also be interpreted as a pulse combining gain. It is an important issue to combine these N_f pulses optimally in the presence of multiple access interference (MAI).

In a conventional matched filter (MF) implementation [2], the receiver weights all the samples from the pulses of the received signal equally. This is the optimal scheme in a single user environment. However, in the presence of MAI, SIR values of those N_f pulses, transmitted for an information bit, can be different. Therefore, the equal gain combining scheme (EGC) is no longer optimal when we have information about the interfering users. If the collisions between the pulses of the user of interest and the interfering users can be detected, a blinking receiver (BR) can be employed [6], which assigns zero weight to pulses colliding with those of interfering users, and assigns equal weight to all other pulses. This combining scheme can be useful in the presence of strong interferers. However, neither the conventional MF receiver nor the BR optimally combines the pulses for a given information bit, when the information about the bit energies and codes of the users is available. Therefore, we propose an MMSE combining scheme, which optimally combines the pulses and maximizes the SIR of the system. Also we derive an approximate expression for bit error probability (BEP) of a BR when the number of pulses per information bit, N_f, is large, and obtain approximate conditions, under which the BR has lower probability than the conventional MF receiver.

The remainder of the paper is organized as follows. Section II describes the signal model. The performance of the MF receiver is considered in Section III. Section IV analyzes the BR and compares its performance to the conventional MF receiver. Section V introduces the MMSE receiver. After the simulation studies in Section VI, Section VII concludes the paper.

II. Signal Model

We consider a binary phase-shift keyed (BPSK) random time-hopping impulse-radio (TH-IR) system where the transmitted signal from user k in a K-user setting is represented by the following model:

$$s_{tx}^{(k)}(t)=\sqrt{\frac{E_k}{N_f}}\sum_{j=-\infty}^{\infty}d_j^{(k)}\,b_{\lfloor j/N_f\rfloor}^{(k)}p_{tx}(t-jT_f-c_j^{(k)}T_c),\quad(1)$$

where $p_{tx}(t)$ is the transmitted UWB pulse, E_k is the bit energy of user k, T_f is the average pulse repetition time (also called the "frame" time), N_f is the number of pulses representing one information symbol, and $b_{\lfloor j/N_f\rfloor}^{(k)}\in\{+1,-1\}$

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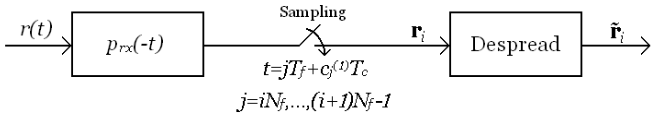

Fig. 1. Sampling and despreading of the received signal for the ith information bit.

is the information symbol transmitted by user k. In order to allow the channel to be exploited by many users and avoid catastrophic collisions, a time-hopping (TH) sequence $\{c_j^{(k)}\}$ is assigned to each user, where $c_j^{(k)} \in \{0, 1, ..., N_c - 1\}$ with equal probability, and $c_j^{(k)}$ and $c_i^{(l)}$ are independent for $(k, j) \neq (l, i)$. This TH sequence provides an additional time shift of $c_j^{(k)} T_c$ seconds to the jth pulse of the kth user where T_c is the chip interval and is chosen to satisfy $T_c \leq T_f / N_c$ in order to prevent the pulses from overlapping. Without loss of generality, $T_f = N_c T_c$ is assumed throughout the paper.

Random polarity codes $d_j^{(k)}$ are binary random variables taking values ± 1 with equal probability and $d_j^{(k)}$ and $d_i^{(l)}$ are independent for $(k, j) \neq (l, i)$ [5].

Using the signal model in (1), the received signal over an additive white Gaussian noise (AWGN) channel in a K-user system can be expressed as

$$\begin{aligned} r(t) &= \sum_{k=1}^{K} \sqrt{\frac{E_k}{N_f}} \sum_{j=-\infty}^{\infty} d_j^{(k)} \, b_{\lfloor j/N_f \rfloor}^{(k)} \\ &\times \; p_{rx}(t - jT_f - c_j^{(k)} T_c) + \sigma_n n(t), \end{aligned} \quad (2)$$

where $p_{rx}(t)$ is the received UWB pulse with unit energy and $n(t)$ is a zero mean white Gaussian noise process with unit spectral density. Although this channel model is not very realistic for UWB systems, it is an important first step towards understanding of more realistic channels, and also approximates the line-of-sight scenarios.

The received signal passes through an MF matched to the received UWB pulse, $p_{rx}(t)$, and is sampled at time instances where a pulse from the user of interest arrives (Figure 1). Then, the discrete time signal model can be expressed as follows, assuming user 1 as the user of interest without loss of generality[2]:

$$\mathbf{r}_i = \mathbf{S}_i \mathbf{A} \mathbf{b}_i + \mathbf{n}_i, \quad (3)$$

where $\mathbf{r}_i$ is an $N_f \times 1$ vector of samples taken at the output of the matched filter at time instances where pulses of user 1 corresponding to the ith bit are received, $\mathbf{A} = \text{diag}\{\sqrt{E_1/N_f}, \cdots, \sqrt{E_K/N_f}\}$, $\mathbf{b}_i = [b_i^{(1)} \cdots b_i^{(K)}]^T$ and $\mathbf{n}_i \sim \mathcal{N}(\mathbf{0}, \sigma_n^2 \mathbf{I})$. $\mathbf{S}_i$ is the generalized signature matrix for the ith bit, which can be expressed as a summation of the desired signal part (SP) and MAI terms as follows:

$$\mathbf{S}_i = \mathbf{S}_{i,SP} + \mathbf{S}_{i,MAI}, \quad (4)$$

where the elements at row m and column n can be expressed as

$$[\mathbf{S}_{i,SP}]_{mn} = \begin{cases} d_{iN_f+m-1}^{(1)}, & n = 1, \\ 0, & n = 2, \ldots, K, \end{cases} \quad (5)$$

and

$$[\mathbf{S}_{i,MAI}]_{mn} = \begin{cases} 0, & n = 1, \\ d_{iN_f+m-1}^{(n)} \mathrm{I}_{iN_f+m-1}^{(n)}, & n = 2, \ldots, K, \end{cases} \quad (6)$$

where $\mathrm{I}_{iN_f+m-1}^{(n)}$ is an indicator function, which is equal to 1, if there is a collision between the mth pulses of user 1 and user n for the ith information bit, and is zero otherwise.

The received signal $\mathbf{r}_i$ in (3) is despread by the polarity codes of user 1:

$$\tilde{\mathbf{r}}_i = \mathbf{D}_i \mathbf{r}_i, \quad (7)$$

where $\mathbf{D}_i = \text{diag}\{d_{iN_f}^{(1)}, \ldots, d_{(i+1)N_f-1}^{(1)}\}$. A linear receiver uses a linear combination of those despread samples and estimate the information bit as $\hat{b}_i = \text{sign}\{y_i\}$, where y_i is the decision variable given by $y_i = \boldsymbol{\theta}^T \tilde{\mathbf{r}}_i$, with $\boldsymbol{\theta}$ being the vector of combining weights.

III. Conventional Matched Filter Receiver

In this case, all the despread samples are weighted equally and the decision variable is obtained as

$$y_i = \mathbf{1}_{N_f} \tilde{\mathbf{r}}_i, \quad (8)$$

where $\mathbf{1}_{N_f}$ is an $N_f \times 1$ vector of ones and $\tilde{\mathbf{r}}_i$ is the despread samples given by (7).

Using (3)-(7), (8) can be expressed as

$$y_i = b_i^{(1)} \sqrt{E_1 N_f} + a_i + w_i, \quad (9)$$

where the first term is the signal part of the output, a_i is the MAI due to interfering users and w_i is the output noise, which is distributed as $w_i \sim \mathcal{N}(0, N_f \sigma_n^2)$.

The MAI term a_i is the sum of interferences from $K - 1$ interfering users. It is shown in [5] and [7] that a_i converges to $\mathcal{N}\left(0, \frac{1}{N_c} \sum_{k=2}^{K} E_k\right)$ as $N_f \longrightarrow \infty$. Then, the BEP of the system can be approximated, for large N_f, as follows:

$$P_{MF} \approx Q\left(\sqrt{\frac{E_1 N_f}{\frac{1}{N_c} \sum_{k=2}^{K} E_k + N_f \sigma_n^2}}\right). \quad (10)$$

This receiver needs to know the TH sequence and polarity code of the user of interest, user 1. It assumes no information about the interfering users.

IV. Blinking Receiver

Given N_f samples of the despread signal $\tilde{\mathbf{r}}_i$, the BR combines only the samples having no interference from the other users. In other words, its decision is based on pulses, where no collisions occur during the reception of these pulses [6]. The decision variable of the BR can be expressed as

$$y_i = \boldsymbol{\beta}_i^T \tilde{\mathbf{r}}_i, \quad (11)$$

[2] The user index is not shown for simplicity. In other words, $\mathbf{r}_i = \mathbf{r}_i^{(1)}$ and $\mathbf{S}_i = \mathbf{S}_i^{(1)}$.

where

$$[\boldsymbol{\beta}_i]_m = \begin{cases} 1, & [\mathbf{S}_i]_{m,2} = \cdots = [\mathbf{S}_i]_{m,K} = 0, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

The following lemma gives the asymptotic distribution of the decision variable y_i as $N_f \longrightarrow \infty$.

Lemma 4.1: As $N_f \longrightarrow \infty$, $y_i/\sqrt{N_f}$, where y_i is given by (11), is asymptotically normally distributed as

$$\frac{y_i}{\sqrt{N_f}} \sim \mathcal{N}\left(b_i^{(1)} p\sqrt{E_1}\,,\, p\,\sigma_n^2\right), \tag{13}$$

where $p = (1 - 1/N_c)^{K-1}$ is the probability that no collision occurs between a pulse of the user of interest and pulses of interfering users.

Proof: Noting that the BR combines those samples having no interference from other users, the decision variable can be expressed as

$$y_i = \sum_{j=iN_f}^{(i+1)N_f-1} x_j, \tag{14}$$

where $x_j = \left(b_i^{(1)}\sqrt{\frac{E_1}{N_f}} + n_j\right) I_j$ with I_j being the indicator function that is equal to one if there is no collision with the jth pulse of the user of interest and is zero otherwise.

Since there are N_c different positions, in which a pulse can reside, and the TH sequences are uniformly distributed, the probability that there is no collision between a given pulse of the user of interest and the pulses of the interfering users is obtained as $p = (1 - 1/N_c)^{(K-1)}$.

Also note that $\{x_j\}_{j=iN_f}^{(i+1)N_f-1}$ forms an independent identically distributed (i.i.d.) sequence, the mean and variance of which can be obtained as $\mu = b_i^{(1)} p\sqrt{E_1/N_f}$ and $\sigma^2 = p(1-p)E_1/N_f + p\,\sigma_n^2$.

Then, using the central limit theorem [8], we have

$$\sqrt{N_f}(\bar{x} - \mu) \xrightarrow{\mathcal{D}} \mathcal{N}(0, \sigma^2), \tag{15}$$

where $\bar{x}$ denotes the sample mean of the sequence $\{x_j\}_{j=iN_f}^{(i+1)N_f-1}$ and $\xrightarrow{\mathcal{D}}$ denotes convergence in distribution.

From (14) and (15), we get

$$\frac{1}{\sqrt{N_f}} y_i - b_i^{(1)} p\sqrt{E_1} \xrightarrow{\mathcal{D}} \mathcal{N}(0\,,\, p\,\sigma_n^2), \tag{16}$$

as $N_f \longrightarrow \infty$, from which (13) follows. □

Using Lemma 4.1, the BEP can be expressed approximately, for large N_f, as

$$P_{BR,appr} \approx Q\left(\frac{\sqrt{E_1 p}}{\sigma_n}\right). \tag{17}$$

If we compare the approximate BEP expressions for the conventional MF receiver and the BR, we see that the former has a lower probability of error when

$$\frac{1}{N_c}\sum_{k=2}^{K} E_k < N_f \sigma_n^2 \left(\frac{1}{p} - 1\right). \tag{18}$$

In other words, when the total energy of the interfering users is smaller than a certain value, the conventional MF receiver performs better. Because in such a case, the BR ignores some samples including small amount of MAI, even though they could have been used in the decision process, which would result in an increased SIR.

The exact expression for BEP can be obtained as [6]

$$P_{BR,exact} = \sum_{i=0}^{N_f} \binom{N_f}{i} p^i (1-p)^{N_f - i} Q\left(\sqrt{\frac{iE_1}{N_f \sigma_n^2}}\right), \tag{19}$$

where $p = (1 - 1/N_c)^{K-1}$. This easily follows by considering that the total number of pulses that the BR combines, that is, total number of pulses with no collision, forms a binomial random variable $\mathcal{B}(N_f, p)$ [6].

V. MMSE Receiver

Now consider the optimal combining scheme among all linear combiners, with $\boldsymbol{\alpha} = [\alpha_1 \cdots \alpha_{N_f}]^T$ denoting the optimal combining weights. Then, the decision variable for the ith information symbol can be expressed as

$$y_i = \boldsymbol{\alpha}^T \tilde{\mathbf{r}}_i, \tag{20}$$

where $\tilde{\mathbf{r}}_i$ is as given in (7).

Using (3)-(7), we can express y_i as

$$y_i = \boldsymbol{\alpha}^T \left(b_i^{(1)}\sqrt{\frac{E_1}{N_f}} \mathbf{1}_{N_f} + \mathbf{D}_i^{(1)} \mathbf{S}_{i,MAI} \mathbf{A} \mathbf{b}_i + \mathbf{D}_i^{(1)} \mathbf{n}_i\right). \tag{21}$$

Then, the MMSE weights are obtained as [9]

$$\boldsymbol{\alpha}_{MMSE} = \mathbf{R}_{\tilde{\mathbf{w}}}^{-1} \mathbf{1}_{N_f}, \tag{22}$$

where $\mathbf{R}_{\tilde{\mathbf{w}}}$ is the correlation matrix of $\tilde{\mathbf{w}}$ with

$$\tilde{\mathbf{w}} = \mathbf{D}_i^{(1)} \mathbf{S}_{i,MAI} \mathbf{A} \mathbf{b}_i + \mathbf{D}_i^{(1)} \mathbf{n}_i. \tag{23}$$

Using (23) and assuming equiprobable information symbols, $\mathbf{R}_{\tilde{\mathbf{w}}}$ can be expressed as

$$\mathbf{R}_{\tilde{\mathbf{w}}} = \mathrm{E}\{\tilde{\mathbf{w}}\tilde{\mathbf{w}}^T\} = \mathbf{D}_i^{(1)} \mathbf{S}_{i,MAI} \mathbf{A}^2 \mathbf{S}_{i,MAI}^T \mathbf{D}_i^{(1)} + \sigma_n^2 \mathbf{I}. \tag{24}$$

The MMSE combining scheme maximizes the SIR of the system, which can be shown to be given by

$$\text{SIR} = \frac{E_1}{N_f} \mathbf{1}_{N_f}^T \mathbf{R}_{\tilde{\mathbf{w}}}^{-1} \mathbf{1}_{N_f}, \tag{25}$$

which is equal to the sum of elements of $\mathbf{R}_{\tilde{\mathbf{w}}}^{-1}$, multiplied by E_1/N_f.

Note that in order to implement this MMSE receiver, random polarity codes and bit energies of *all* the users must be known. Also the indices of the users colliding with each pulse of the user of interest are required. Moreover, the implementation of the receiver requires a matrix inversion operation. Therefore, the MMSE receiver is more complex than the conventional MF receiver and the BR. However, it serves as a reference for suboptimal linear receivers. Moreover, in some situations, a training sequence can be employed to estimate $\mathbf{R}_{\tilde{\mathbf{w}}}$ in (24) and then the MMSE combining scheme can be employed to obtain better BEP performance.

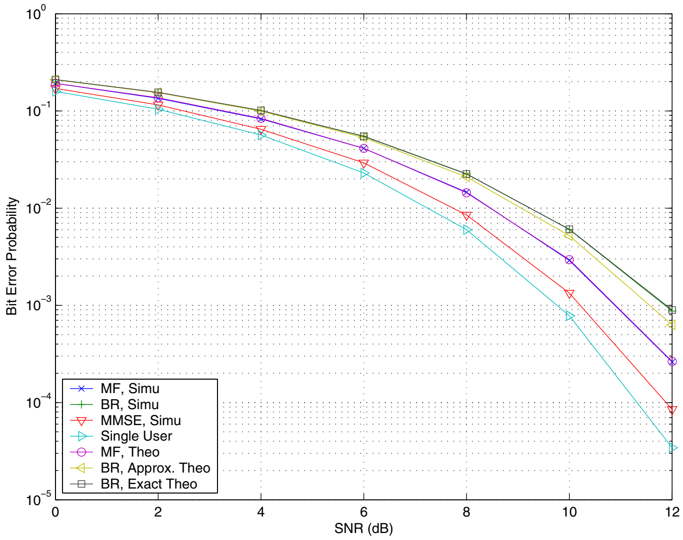

Fig. 2. BEP vs SNR for a 5-user system with $N_f = 25$, $N_c = 10$, $\sigma_n^2 = 0.1$ and $E_k = 2$ for $k = 2, 3, 4, 5$.

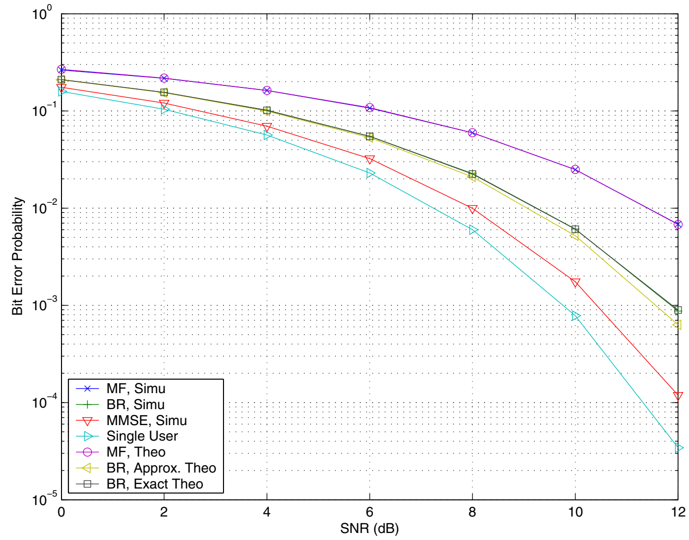

Fig. 3. BEP vs SNR for a 5-user system with $N_f = 25$, $N_c = 10$, $\sigma_n^2 = 0.1$ and $E_k = 10$ for $k = 2, 3, 4, 5$.

VI. Simulation Results

In this section, the performances of the three types of receivers considered in the previous sections are compared in a 5-user TH-IR system. The system parameters are chosen as $N_f = 25$ and $N_c = 10$ and the noise variance is equal to 0.1, that is, $\sigma_n^2 = 0.1$.

Figure 2 plots the BEP versus SNR (SNR= $10 \log \frac{E_1}{\sigma_n^2}$), where the bit energies of the interfering users are equal to 2. From the plot, it is observed that the BR has the highest BEP, which can be also checked by the approximate condition in (18). The MMSE receiver performs the best as expected and it is close to the single user bound. For the conventional MF receiver, the approximate theoretical results agree well with the simulation results. For the BR, the exact theoretical results are closer to the simulation results compared to the approximate theoretical results for large SNR values. This is because N_f is not large enough to ignore the first term of the variance σ^2 in the proof of Lemma 4.1 for large SNR values.

Figure 3 plots the BEP versus SNR, where the bit energy of each interfering user is equal to 10. From the plot, it is observed that the performance of the BR is the same as in Figure 2 since it only combines those samples which has no interference from the other users. However, the performance of the MF receiver is degraded in this strong MAI scenario, and it performs worse than the BR. The degradation of the MMSE receiver is very small and its performance is still quite close to the performance of an MF in a single-user system.

VII. Conclusions

We have considered three different types of pulse combining schemes. The first scheme gives equal weight to each pulse, which is employed in a conventional matched filter receiver. This scheme is the simplest one since it does not need any information about interfering users. The second pulse combining scheme is employed by a blinking receiver, which assigns the same weight to all the pulses received with no interference from other users and assigns zero weight to the pulses colliding with those of any other users. This scheme requires detection of collisions between the pulses of the user of interest and those of the interfering users. We have shown that if the power of the interferers is higher than a certain value, then the BR has a lower BEP than the conventional MF receiver. Finally, we have considered an MMSE pulse combining scheme, which optimally combines the samples from different pulses. This receiver is the best among the three receivers considered. However, its complexity is considerably higher than the first two receivers.

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