

# Impulse Radio Systems with Multiple Types of Ultra-Wideband Pulses<sup>0</sup>

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**Abstract** — Spectral properties and performance of multi-pulse impulse radio ultra-wideband systems with pulse-based polarity randomization are analyzed. Instead of a single type of pulse transmitted in each frame, multiple types of pulses are considered, which is shown to reduce the effects of multiple-access interference. First, the spectral properties of a multi-pulse impulse radio system is investigated. It is shown that the power spectral density is the average of spectral contents of different pulse shapes. Then, approximate closed-form expressions for bit error probability of a multi-pulse impulse radio system are derived for RAKE receivers in asynchronous multiuser environments. The theoretical and simulation results indicate that impulse radio systems that are more robust against multiple-access interference than a “classical” impulse radio system can be designed with multiple types of ultra-wideband pulses.

**Index Terms**— Ultra-wideband (UWB), multi-pulse impulse radio (IR), RAKE receivers, multiple-access interference (MAI), asynchronous systems.

## I. INTRODUCTION

Impulse radio ultra-wideband (IR-UWB) systems hold great promise for a variety of applications such as short-range high-speed data transmission and precise location estimation. In such systems, a train of pulses is transmitted and information is usually conveyed by the position or the polarity of the pulses [1]–[6]. In order to prevent catastrophic collisions among different users and thus provide robustness against multiple-access interference (MAI), each information symbol is represented by a sequence of pulses; the positions of the pulses within that sequence are determined by a pseudo-random time-hopping (TH) sequence specific to each user [1].

In “classical” impulse radio, a single type of UWB pulse is transmitted in all frames of all the users [1]. In asynchronous multiuser environments, the autocorrelation function of the pulse becomes an important factor in determining the effects of the MAI [7]. In order to reduce those effects, UWB pulses with fast decaying autocorrelation functions are desirable. However, such autocorrelation functions also result in a considerable decrease in the desired signal part of the receiver output in the presence of timing jitter [8]. Moreover, when there is an exact overlap between the pulses of two users, the MAI is usually very significant. Hence, there is not much flexibility in choosing the pulse shape in order to combat against interference effects. However, in IR systems with multiple types of UWB pulses, MAI can be mitigated by means

of different types of UWB pulses with good cross-correlation properties. Multi-pulse IR systems have recently been proposed in [9]. However, there has been no theoretical analysis of those systems, in terms of their spectral properties and bit error probability (BEP) performance, and no quantitative investigation of the gains that can be obtained by multiple types of UWB pulses.

In this paper, we consider an asynchronous multiuser environment and analyze the BEP performance of a generic RAKE receiver over frequency-selective channels. The results are valid for different number of UWB pulse types, hence cover the single-pulse system as a special case. We also analyze average power spectrum density (PSD) of multi-pulse IR signals, and obtain a simple relation between the Fourier transforms of the UWB pulses and the average PSD of the transmitted signal.

The remainder of the paper is organized as follows. Section II describes the transmitted signal model and Section III analyzes its spectral properties. In Section IV, the performance analysis of multi-pulse IR RAKE receivers is presented. The simulation results are given in Section V, which is followed by the concluding remarks in Section VI.

## II. SIGNAL MODEL

Consider a  $K$ -user environment with the following transmitted signal from user  $k$ :

$$s^{(k)}(t) = \frac{1}{\sqrt{N_f}} \sum_{j=-\infty}^{\infty} d_j^{(k)} b_{\lfloor j/N_f \rfloor}^{(k)} p_j^{(k)}(t - jT_f - c_j^{(k)}T_c), \quad (1)$$

where  $p_j^{(k)}(t)$  is the UWB pulse transmitted in the  $j$ th frame of user  $k$ ,  $b_{\lfloor j/N_f \rfloor}^{(k)} \in \{+1, -1\}$  is the equiprobable information bit,  $T_f$  is the frame time,  $T_c$  is the chip interval, and  $N_f$  is the number of frames/pulses per information symbol. The time hopping (TH) code, denoted by  $c_j^{(k)}$ , is modelled by a uniform distribution in  $\{0, 1, \dots, N_c - 1\}$ , with  $N_c = T_f/T_c$  being the number of chips per frame, and  $c_j^{(k)}$  and  $c_i^{(l)}$  are independent for  $(j, k) \neq (i, l)$ . Random polarity codes,  $d_j^{(k)}$ , are binary random variables taking values  $\pm 1$  with equal probability, and  $d_j^{(k)}$  and  $d_i^{(l)}$  are independent for  $(j, k) \neq (i, l)$  [10]. Use of random polarity codes helps reduce the spectral lines in the power spectral density of the transmitted signal [11] and mitigate the effects of MAI [10]. The receiver for user  $k$  is assumed to know its polarity code.

Note the difference of the signal model in (1) from a classical IR system, in which the same pulse is used in all the frames. In other words, the signal model in (1) is a more general formulation of an IR system, which reduces to the original proposal in [1] when  $p_j^{(k)}(t) = p(t) \forall j, k$ . We assume that there are  $N_p$  different types of pulses employed in the system and  $p_{j+iN_p}^{(k)}(t) = p_j^{(k)}(t)$  for any integer  $i$ . Also, for simplicity of

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the expressions, we assume that  $N_f$  is an integer multiple of  $N_p$ .

### III. POWER SPECTRUM DENSITY ANALYSIS

In order to evaluate the spectral properties of the transmitted signal, its (average) PSD needs to be calculated. Therefore, we first calculate the autocorrelation function of  $s(t)$  in (1) as follows<sup>3</sup>:

$$\begin{aligned} \phi_{ss}(t + \tau, t) &= \text{E}\{s(t + \tau)s(t)\} = \\ &= \frac{1}{N_f} \sum_{j=-\infty}^{\infty} \text{E}\{p_j(t + \tau - c_j T_c - j T_f) p_j(t - c_j T_c - j T_f)\}, \end{aligned} \quad (2)$$

where we employ the fact that the random polarity codes are i.i.d. random variables taking  $\pm 1$  with equal probability.

From (2), it is observed that  $s(t)$  is not wide-sense stationary (WSS) since the autocorrelation function is not independent of  $t$ . However, note that  $s(t)$  is a zero mean cyclostationary process [13] since  $\phi_{ss}(t + \tau, t)$  is periodic with a period of  $N_p T_f$ . Therefore, we can obtain the time-average autocorrelation function as

$$\begin{aligned} \bar{\phi}_{ss}(\tau) &= \frac{1}{N_p T_f} \int_0^{N_p T_f} \phi_{ss}(t + \tau, t) dt \\ &= \frac{1}{N_p T_f N_f} \sum_{l=0}^{N_p-1} \int_{-\infty}^{\infty} p_l(t + \tau) p_l(t) dt, \end{aligned} \quad (3)$$

the Fourier transform of which gives the average PSD as follows:

$$\Phi_{ss}(f) = \frac{1}{N_p T_s} \sum_{l=0}^{N_p-1} |P_l(f)|^2, \quad (4)$$

where  $T_s = N_f T_f$  is the symbol interval, and  $P_l(f)$  is the Fourier transform of  $p_l(t)$ .

Note from (4) that the average PSD of the signal is the average value of the squares of the Fourier transforms of the pulses. The dependence on the pulse spectra only is the result of the pulse-based polarity randomization [11], [12]. Moreover, note that there can be flexibility in shaping the PSD by proper choice of UWB pulse types.

### IV. PERFORMANCE ANALYSIS

Consider the following channel model for user  $k$ :

$$h^{(k)}(t) = \sum_{l=0}^{L-1} \alpha_l^{(k)} \delta(t - \tau_l^{(k)}), \quad (5)$$

where  $\alpha_l^{(k)}$  and  $\tau_l^{(k)}$  are the fading coefficient and the delay for the  $l$ th path of user  $k$ .

Using the channel model in (5) and the transmitted signal in (1), the received signal can be expressed as

$$\begin{aligned} r(t) &= \sum_{k=1}^K \frac{1}{\sqrt{N_f}} \sum_{j=-\infty}^{\infty} d_j^{(k)} b_{\lfloor j/N_f \rfloor}^{(k)} u_j^{(k)}(t - j T_f - c_j^{(k)} T_c \\ &\quad - \tau_0^{(k)}) + \sigma_n n(t), \end{aligned} \quad (6)$$

<sup>3</sup>We drop the user index  $k$  in this section, for notational convenience.

with

$$u_j^{(k)}(t) \triangleq \sum_{l=0}^{L-1} \alpha_l^{(k)} w_j^{(k)}(t - \tau_l^{(k)} + \tau_0^{(k)}), \quad (7)$$

where  $w_j^{(k)}(t)$  is the received UWB pulse in the  $j$ th frame of user  $k$ , and  $n(t)$  is a zero mean white Gaussian process with unit spectral density.

We consider a generic RAKE receiver that can represent different combining schemes, such as equal gain combining or maximal ratio combining. It can be expressed as the correlation of the received signal in (6) with the following template signal for the  $i$ th information bit, where we consider the user 1 as the user of interest without loss of generality:

$$s_i^{(1)}(t) = \sum_{j=i N_f}^{(i+1)N_f-1} d_j^{(1)} v_j^{(1)}(t - j T_f - c_j^{(1)} T_c), \quad (8)$$

with

$$v_j^{(1)}(t) \triangleq \sum_{l=0}^{L-1} \beta_l w_j^{(1)}(t - \tau_l^{(1)}), \quad (9)$$

where  $\beta_l$  denotes the RAKE combining coefficient for the  $l$ th path. We assume  $\tau_0^{(1)} = 0$  without loss of generality. Note that for a partial or selective RAKE receiver [14], the combining coefficients for those paths that are not utilized are set to zero.

We assume that the TH codes are constrained to the set  $\{0, 1, \dots, N_h - 1\}$ , where  $N_h T_c + \tau_{L-1}^{(k)} < T_f \forall k$ , so that there is no inter-frame interference (IFI). Then, using (6) and (8), the decision variable for detecting the  $i$ th bit of user 1 can be obtained as:

$$\begin{aligned} Y_i &= \int r(t) s_i^{(1)}(t) dt \\ &= b_i^{(1)} \frac{1}{\sqrt{N_f}} \sum_{j=i N_f}^{(i+1)N_f-1} \phi_{u_j^{(1)} v_j^{(1)}}(0) + M_i + N_i, \end{aligned} \quad (10)$$

with

$$\phi_{u_i^{(k)} v_j^{(l)}}(x) \triangleq \int u_i^{(k)}(t - x) v_j^{(l)}(t) dt, \quad (11)$$

where the first term in (10) is the desired signal part of the output,  $M_i$  is the MAI, and  $N_i$  is the output noise for the  $i$ th information bit. For simplicity of the expressions, we drop the bit index  $i$  and consider the 0th bit without loss of generality, for the rest of the analysis.

#### IV.A MULTIPLE-ACCESS INTERFERENCE

Consider the MAI term  $M$  in (10), which is the sum of interference terms from  $(K - 1)$  users,  $M = \frac{1}{\sqrt{N_f}} \sum_{k=2}^K M^{(k)}$ , where  $M^{(k)}$  can be expressed as

$$M^{(k)} = \sum_{j=0}^{N_f-1} \hat{M}_j^{(k)}, \quad (12)$$

with  $\hat{M}_j^{(k)}$  denoting the MAI from user  $k$  to the  $j$ th frame of the first user. From (6), (8) and (11),  $\hat{M}_j^{(k)}$  can be expressed

as

$$\hat{M}_j^{(k)} = d_j^{(1)} \sum_{m=-\infty}^{\infty} d_m^{(k)} b_{[m/N_f]}^{(k)} \phi_{u_m^{(k)} v_j^{(1)}} \left( (m-j)T_f + (c_m^{(k)} - c_j^{(1)})T_c + \tau_0^{(k)} \right), \quad (13)$$

where  $\tau_0^{(k)}$  denotes the amount of asynchronism between user  $k$  and the user of interest, user 1, since we assume  $\tau_0^{(1)} = 0$ . In practical situations, the users in an IR-UWB system are not synchronized. For example, in IEEE 802.15.3a personal area networks, the MAI comes from neighboring uncoordinated piconets. Therefore, we consider an asynchronous scenario in which the amount of asynchronism is uniformly distributed in a symbol interval; that is  $\tau_0^{(k)} \sim \mathcal{U}[0, N_f T_f] \forall k$ .

In order to obtain an approximate closed form expression for the BEP, we assume large number of interferers with equal received powers (perfect power control), and approximate the total MAI by a zero mean Gaussian random variable using the standard Gaussian approximation (SGA). Although this assumption may not be very realistic in some situations, it will give us an idea about the gains that can be obtained by multiple types of UWB pulses. Moreover, if the number of users is not very large and the received powers are unbalanced, we can employ the Gaussian approximation in [7] in order to obtain an approximate BEP expression.

In order to calculate the variance of  $M$  in (10), we first consider that of  $M^{(k)}$  in (12). Note that  $E\{\hat{M}_j^{(k)} \hat{M}_l^{(k)}\} = 0$  for  $j \neq l$  due to the random polarity codes. Therefore,  $E\{(M^{(k)})^2\} = \sum_{j=0}^{N_f-1} E\{(\hat{M}_j^{(k)})^2\}$ .

From (13), the variance of  $\hat{M}_j^{(k)}$  can be obtained as follows, after averaging over polarity randomization and TH codes, and the delay of user  $k$ :

$$E\{(\hat{M}_j^{(k)})^2\} = \frac{\sigma_M^2(k, j)}{N_h^2}, \quad (14)$$

where

$$\sigma_M^2(k, j) \triangleq \frac{1}{T_f N_p} \sum_{m=j-N_p}^j \sum_{l=1-N_h}^{N_h-1} (N_h - |l|) \int_0^{N_p T_f} \phi_{u_m^{(k)} v_j^{(1)}}^2 \left( (m-j)T_f + lT_c + \tau_0^{(k)} \right) d\tau_0^{(k)}. \quad (15)$$

Note that since there are  $N_p$  different pulse shapes, it is enough to integrate over  $N_p$  frames, instead of the whole symbol period.

For the classical IR system, where a single UWB pulse  $w_0(t)$  is employed, the result reduces, after some manipulation, to

$$E\{(\hat{M}_j^{(k)})^2\} = \frac{1}{T_f N_h^2} \sum_{l=1-N_h}^{N_h-1} (N_h - |l|) \int_{-T_f}^{T_f} \phi_{u_0^{(k)} v_0^{(1)}}^2 \left( lT_c + \tau_0^{(k)} \right) d\tau_0^{(k)}. \quad (16)$$

Note from (15) and (16) that the MAI term for the classical IR system depends on the autocorrelation function of the UWB pulse, whereas that for the multi-pulse IR system depends on both the autocorrelation and the cross-correlation functions of the pulses, which provides flexibility to design pulses with good cross-correlation properties in order to reduce the effects of MAI.

## IV.B OUTPUT NOISE

The noise  $N$  in (10) is distributed as  $\mathcal{N}\left(0, \sigma_n^2 \int |s_i^{(1)}(t)|^2 dt\right)$ . Using the expression in (8) for  $s_i^{(1)}(t)$ , we can obtain the distribution of  $N$  for an IR system with  $N_p$  different UWB pulses as follows:

$$N \sim \mathcal{N}\left(0, \sigma_n^2 \frac{N_f}{N_p} \sum_{j=0}^{N_p-1} \phi_{v_j^{(1)}}(0)\right), \quad (17)$$

where  $\phi_{v_j^{(k)}}(x) \triangleq \int v_j^{(k)}(t-x)v_j^{(k)}(t)dt$  is the autocorrelation function of  $v_j^{(k)}(t)$ .

## IV.C BIT ERROR PROBABILITY

Using the results in the previous sections, we obtain the approximate BEP expression for an IR system employing  $N_p$  different UWB pulses as follows:

$$P_e \approx Q\left(\frac{\frac{1}{\sqrt{N_p}} \sum_{j=0}^{N_p-1} \phi_{u_j^{(1)} v_j^{(1)}}(0)}{\sqrt{\frac{1}{N_f N_h^2} \sum_{j=0}^{N_p-1} \sum_{k=2}^K \sigma_M^2(k, j) + \sigma_n^2 \sum_{j=0}^{N_p-1} \phi_{v_j^{(1)}}(0)}}\right) \quad (18)$$

for large  $K$ , where  $\sigma_M^2(k, j)$  is as given in (15).

For the case of a single type of UWB pulse  $w_0(t)$ , the BEP can be expressed as

$$P_e \approx Q\left(\frac{\phi_{u_0^{(1)} v_0^{(1)}}(0)}{\sqrt{\frac{1}{N_f N_h^2} \sum_{k=2}^K \sigma_M^2(k) + \sigma_n^2 \phi_{v_0^{(1)}}(0)}}\right), \quad (19)$$

where

$$\sigma_M^2(k) \triangleq \frac{1}{T_f} \sum_{l=1-N_h}^{N_h-1} (N_h - |l|) \int_{-T_f}^{T_f} \phi_{u_0^{(k)} v_0^{(1)}}^2 \left( lT_c + \tau_0^{(k)} \right) d\tau_0^{(k)}.$$

From (18) and (19), the improvements in BEP as a result of the use of multiple UWB pulse types can be quantified approximately.

## V. SIMULATION RESULTS

In this section, we compare BEP performances of a single pulse and a double-pulse IR system. In the double-pulse system, each user transmits the 4th and 5th order modified Hermite pulses (MHPs) [9] alternately, whereas the single-pulse system employs the 4th order MHP in all the frames.

The systems parameters are  $K = 20$  users,  $N_f = 2$  frames per symbol,  $N_c = 40$  chips per frame,  $T_c = 1$  ns, and  $N_h = 3$ . We consider a scenario, where the received energy of the interferers is 5 times larger than that of the user of interest. All the channels have  $L = 20$  taps, which are generated independently according to a channel model with exponentially decaying ( $E\{|\alpha_l|^2\} = \Omega_0 e^{-\lambda l}$ ) and log-normally fading ( $|\alpha_l| \sim \mathcal{LN}(\mu_l, \sigma^2)$ ) channel amplitudes, random signs for channel taps, and exponential distribution for the path arrivals with a mean

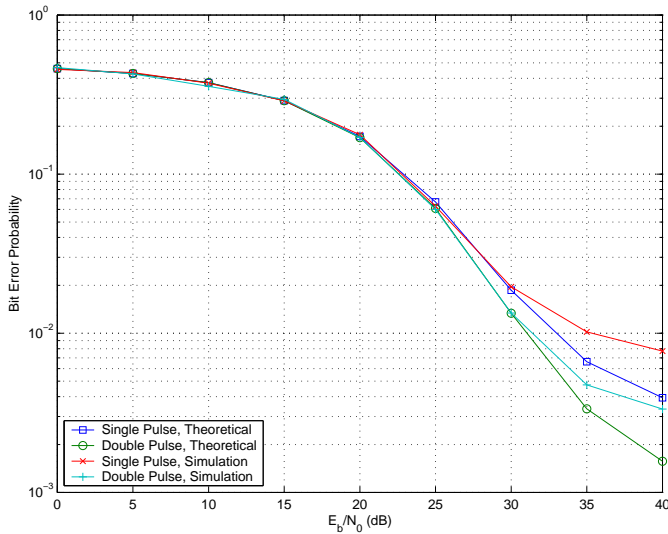


Figure 1: BEP versus  $E_b/N_0$  for single and double-pulse IR systems.

$\hat{\mu}$ . The channel parameters are  $\lambda = 0.5$ ,  $\sigma^2 = 1$ , and  $\hat{\mu} = 1.5$  ns, and  $\mu_l$  can be calculated from  $\mu_l = 0.5 \left[ \ln\left(\frac{1-e^{-\lambda}}{1-e^{-\lambda L}}\right) - \lambda - 2\sigma^2 \right]$ , for  $l = 0, 1, \dots, L - 1$ .

Figure 1 shows the BEP performance of the all-RAKE receivers [14] for the single and double-pulse systems. Both the theoretical and the simulation results are shown, which are in a quite good agreement, except for high SNR values, where the SGA gives optimistic evaluation. From the plot, the double-pulse system is observed to have a better performance than the single-pulse system. From the expressions in Section IV, the amount of MAI to the double-pulse system can be calculated to be %20 smaller than that of the single-pulse system. We can obtain further gains by using more UWB pulse types and/or MHPs that are several orders apart [9].

## VI. CONCLUSIONS

We have obtained closed-form expressions for the average PSD and approximate BEP of multi-pulse IR systems with pulse-based polarity randomization. We have shown that by using different types of UWB pulses, the effects of MAI can be mitigated, and we have quantified the performance gains by using the approximate BEP expressions.

The future work includes a more detailed performance analysis of the multi-pulse IR system in the presence of IFI, considering the effects of the MAI using more accurate approximations that do not require large number of users or equal energy interferers.

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