The Trade-off Between Processing Gains of Impulse Radio Systems in the Presence of Timing Jitter[1]

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***Abstract*— In time hopping impulse radio, N_f pulses of duration T_c are transmitted for each symbol. This gives rise to two types of processing gain: (i) pulse combining gain, which is a factor N_f, and (ii) pulse spreading gain, which is $N_c = T_f/T_c$, where T_f is the mean interval between two subsequent pulses. This paper investigates the trade-off between these two types of processing gain with and without random polarity codes in the presence of timing jitter. Approximate expressions for bit error probability are derived for both coded and uncoded systems over additive white Gaussian noise channels and are used as the criterion to choose optimal N_f and N_c values. The effects of timing jitter and multiple access interference on the selection of optimal system parameters are explained through theoretical analysis. Simulation studies support the theoretical results.**

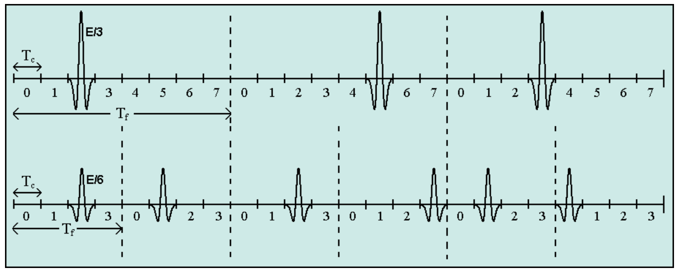

Fig. 1. Two different cases for an uncoded BPSK-modulated TH-IR system when $N = 24$. For the first case, $N_c = 8$, $N_f = 3$, pulse energy is $E/3$ and for the second case $N_c = 4$, $N_f = 6$, pulse energy is $E/6$.

I. Introduction

Recently, communication systems that employ ultra-wideband (UWB) signals have drawn considerable attention. UWB systems occupy a bandwidth larger than 500 MHz; due to the large spreading factors and low power spectral densities, they can coexist with incumbent systems in the same frequency range. Recent Federal Communications Commission (FCC) rulings [6, 7] specify the regulations for UWB communication systems in the US. Similar rulings are expected in the near future for Europe and Japan.

Commonly, impulse radio (IR) systems, which transmit very short pulses with a low duty cycle, are employed to implement UWB systems ([1]-[2]). In an IR system, a train of pulses is sent and information is usually conveyed by the position or the polarity of the pulses, which correspond to Pulse Position Modulation (PPM) and Binary Phase Shift Keying (BPSK), respectively. Also, in order to prevent catastrophic collisions among different users and thus provide robustness against multiple access interference, each information symbol is represented not by one pulse but by a sequence of pulses, and the location of the pulses within the sequence is determined by a pseudo-random time-hopping (TH) sequence [1]. For example, the first signal in Figure 1 is an uncoded[4] BPSK-modulated TH-IR signal where three pulses represent one bit and the pulse positions are determined by the TH sequence $\{2, 5, 3\}$. (A binary information bit of $+1$ is shown in the figure. A binary information bit of -1 corresponds to the case where the polarity of all the pulses is flipped.)

The number of pulses that are sent for each information symbol is denoted by N_f. At the receiver end, these N_f pulses are properly combined to improve signal-to-noise ratio (SNR). This type of processing gain is called the pulse combining gain. The second type of processing gain N_c is the pulse spreading gain and is defined as the ratio of average time between two consecutive transmissions and the actual transmission time, that is, $N_c = T_f/T_c$. The total processing gain is defined as $N = N_c N_f$ and assumed to be a large constant number [5]. The aim of this paper is to investigate the trade-off between the two types of processing gain, N_c and N_f, and to find an optimal N_c (N_f) value such that the bit error probability (BEP) of the system is minimized[5]. In other words, the problem is to decide whether or not sending more pulses each with less energy is more desirable in terms of BEP performance than sending fewer pulses each with more energy (Figure 1).

The trade-offs between these two types of processing gain was originally investigated in [5], where it was concluded that in multiuser flat fading channels, the system performance is independent of the pulse combining gain for a coded system and it is in favor of smaller pulse combining gain for an uncoded system. However, no timing jitter was considered in that work. As we will see in this paper, timing jitter can have a significant effect on the trade-off between the processing gains, which modifies the dependency of the BEP expressions

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[4]In coded systems, the polarity of all pulses are determined by a random polarity code sequence and a binary information bit, as will be explained later.

[5]The FCC also imposes restriction on peak-to-average ratio (PAR), which is not considered in this paper.

on processing gain parameters. Thus, in this paper, the trade-off between two types of processing gain is investigated in the presence of timing jitter and approximate expressions for BEP are derived for both coded and uncoded systems.

The remainder of this paper is organized as follows. Section II describes the transmitted signal model and components of the received signal at the output of a matched filter (MF) receiver. The BEP expressions for coded and uncoded systems are derived in Sections III and IV, respectively and the trade-off between the processing gains is investigated. Section V presents some simulation studies and numerical examples and finally Section VI concludes the paper.

II. Signal Model

Consider a BPSK random time-hopping impulse-radio (TH-IR) system where the transmitted signal from user k in an N_u-user setting is represented by the following model:

$$s_{tx}^{(k)}(t) = \sqrt{\frac{E_k}{N_f}} \sum_{j=-\infty}^{\infty} d_j^{(k)}\, b_{\lfloor j/N_f \rfloor}^{(k)} w_{tx}(t - jT_f - c_j^{(k)}T_c - \epsilon_j^{(k)}), \quad (1)$$

where w_{tx} is the transmitted UWB pulse, E_k is the bit energy of user k, $\epsilon_j^{(k)}$ is the timing jitter at jth pulse of the kth user, T_f is the average pulse repetition time (also called the "frame" time), N_f is the number of pulses representing one information symbol, which is called the pulse combining gain, and $b_{\lfloor j/N_f \rfloor}^{(k)} \in \{+1, -1\}$ is the information symbol transmitted by user k. In order to allow the channel to be exploited by many users and avoid catastrophic collisions, a TH sequence $\{c_j^{(k)}\}$ is assigned to each user, where $c_j^{(k)} \in \{0, 1, ..., N_c - 1\}$ with uniform probability, and $c_j^{(k)}$ and $c_i^{(l)}$ are independent for $(k, j) \neq (l, i)$. This TH sequence provides an additional time shift of $c_j^{(k)} T_c$ seconds to the jth pulse of the kth user where T_c is the chip interval and is chosen to satisfy $T_c \leq T_f / N_c$ in order to prevent the pulses from overlapping. Without loss of generality, $T_f = N_c T_c$ is assumed throughout the paper.

Two different IR systems are considered depending on $d_j^{(k)}$. Complying with the terminology established in [5], the system will be called "uncoded" if $d_j^{(k)} = 1$, $\forall k, j$, and it will be called "coded" if $d_j^{(k)}$ are binary random variables taking values ± 1 with equal probability and are independent for $(k, j) \neq (l, i)$. The first type of system is the original proposal for transmission over UWB channels ([1], [8]) while a version of the second type is proposed in [9].

The timing jitter $\epsilon_j^{(k)}$ in (1) mainly represents the inaccuracies of the local pulse generators at the transmitters and are modelled as being independent and identically distributed (i.i.d.) among the pulses of a given user. That is, $\epsilon_j^{(k)}$ for $j = \ldots, -1, 0, 1, \ldots$ form an i.i.d. sequence for each k. Also the jitter is assumed to be smaller than the chip interval T_c, that is, $\max_{j,k} |\epsilon_j^{(k)}| < T_c$, which is usually the case for practical situations.

The parameter $N = N_c N_f$ is defined to be the total processing gain. Assuming a large and constant N value, the aim is to obtain the optimal N_c (N_f) value that minimizes the BEP of the system.

The received signal over an additive white Gaussian noise (AWGN) channel in an N_u-user system can be expressed as

$$r(t) = \sum_{k=1}^{N_u} \sqrt{\frac{E_k}{N_f}} \sum_{j=-\infty}^{\infty} d_j^{(k)}\, b_{\lfloor j/N_f \rfloor}^{(k)} \times w_{rx}(t - jT_f - c_j^{(k)} T_c - \epsilon_j^{(k)}) + \sigma_n n(t), \quad (2)$$

where w_{rx} is the received UWB pulse with unit energy and $n(t)$ is a zero mean white Gaussian noise with unit spectral density. Even though this channel model is not very realistic for UWB systems, it is an important first step towards understanding of a real system since the main ideas in the analysis can be extended to multipath scenarios, which are not considered here due to space limitations (see [4] for extensions to multipath channels).

For analytical tractability, we assume that users are symbol-synchronized. In fact, for coded systems, the system performance is the same whether symbol synchronization or chip synchronization is assumed (see [3]). Generally, assuming synchronization among different users in a random TH-IR system increases the effect of MAI and serves as a worst-case scenario, as studied in [4] and [3].

Considering a MF receiver, the template signal at the receiver can be expressed as follows:

$$s_{temp}^{(1)}(t) = \frac{1}{\sqrt{N_f}} \sum_{j=iN_f}^{(i+1)N_f - 1} d_j^{(1)} w_{rx}(t - jT_f - c_j^{(1)} T_c), \quad (3)$$

where, without loss of generality, user 1 is assumed to be the user of interest and the multiplication by $1/\sqrt{N_f}$ is used just for scaling purposes, which, of course, does not affect the BEP expression. Also note that no timing jitter is considered for the template signal since the jitter model in the received signal account for that jitter as well.

From (2) and (3), the MF output for user 1 can be expressed as follows[6]:

$$y_1 = \frac{\sqrt{E_1}}{N_f}\, b_i^{(1)} \sum_{j=iN_f}^{(i+1)N_f - 1} R(\epsilon_j^{(1)}) + a + n, \quad (4)$$

where the first term is the signal part of the output, with $R(x) = \int_{-\infty}^{\infty} w_{rx}(t) w_{rx}(t - x) dt$ being the autocorrelation function of the UWB pulse, a is the multiple access interference (MAI) due to other users and n is the output noise, $n \sim \mathcal{N}(0, \sigma_n^2)$.

The MAI term can be expressed as a sum of interference terms from each user, that is, $a = \sum_{k=2}^{N_u} \frac{\sqrt{E_k}}{N_f} a^{(k)}$, where each interference term is in turn the summation of interference term

[6]The self-interference term due to timing jitter is ignored since it becomes negligible for large N_c and/or small $\mathrm{E}\{R^2(T_c - |\epsilon^{(1)}|)\}$ values, where $R(x) = \int_{-\infty}^{\infty} w_{rx}(t) w_{rx}(t - x) dt$.

each of which affects one pulse of the template signal:

$$a^{(k)} = \sum_{l=iN_f}^{(i+1)N_f-1} a_l^{(k)}, \tag{5}$$

where

$$a_l^{(k)} = d_l^{(1)} \int w_{rx}(t - lT_f - c_l^{(1)}T_c) \times \sum_{j=-\infty}^{\infty} d_j^{(k)} b_{\lfloor j/N_f \rfloor}^{(k)} w_{rx}(t - jT_f - c_j^{(k)}T_c + \epsilon_j^{(k)})\, dt. \tag{6}$$

As can be seen from (6), $a_l^{(k)}$ determines the interference from user k to the lth pulse of the template signal.

Let $p_l^{(1)}$ denote the position of the lth pulse of the template signal in the lth frame ($p_l^{(1)} = 1, ..., N_c$) for $l = iN_f, ..., (i+1)N_f - 1$. Similarly, write $p_l^{(k)}$ for the position of the lth pulse of the received signal from user k. Then, $a_l^{(k)}$ can be expressed for $p_l^{(1)} = 2, ..., N_c - 1$ as follows:

$$\begin{aligned} a_l^{(k)} = b_i^{(k)} d_l^{(1)} d_l^{(k)} [&R(\epsilon_l^{(k)})\, I_{\{p_l^{(1)} = p_l^{(k)}\}} \\ &+ R(T_c - \epsilon_l^{(k)})\, I_{\{p_l^{(1)} - p_l^{(k)} = 1\}} I_{\{\epsilon_l^{(k)} > 0\}} \\ &+ R(T_c + \epsilon_l^{(k)})\, I_{\{p_l^{(k)} - p_l^{(1)} = 1\}} I_{\{\epsilon_l^{(k)} < 0\}}], \end{aligned} \tag{7}$$

for $l = iN_f, ..., (i+1)N_f - 1$, where I_A is the indicator function taking value 1 in set A and 0 outside. In obtaining (7), the following observation is employed: There occurs interference from user k to the lth pulse of the template signal if user k has its lth pulse at the same position as the lth pulse of the template signal or it has its lth pulse at a neighboring position to lth pulse of the template signal and there is a partial overlap due to the effect of timing jitter.

For $p_l^{(1)} = 1$, we also consider the interference from the previous frame of the signal received from user k:

$$\begin{aligned} a_l^{(k)} = b_i^{(k)} d_l^{(1)} d_l^{(k)} [&R(\epsilon_l^{(k)}) I_{\{p_l^{(k)} = 1\}} \\ &+ R(T_c + \epsilon_l^{(k)}) I_{\{p_l^{(k)} = 2\}} I_{\{\epsilon_l^{(k)} < 0\}}] \\ + b_i^{(k)} d_l^{(1)} d_{l-1}^{(k)} &R(T_c - \epsilon_{l-1}^{(k)}) I_{\{p_{l-1}^{(k)} = N_c\}} I_{\{\epsilon_{l-1}^{(k)} > 0\}}, \end{aligned} \tag{8}$$

for $l = iN_f + 1, ..., (i+1)N_f - 1$. Note that for $l = iN_f$, we just need to replace $b_i^{(k)}$ in the third term by $b_{i-1}^{(k)}$ since the previous bit will be in effect in that case.

Similarly, for $p_l^{(1)} = N_c$,

$$\begin{aligned} a_l^{(k)} = b_i^{(k)} d_l^{(1)} d_l^{(k)} [&R(\epsilon_l^{(k)}) I_{\{p_l^{(k)} = N_c\}} \\ &+ R(T_c - \epsilon_l^{(k)}) I_{\{p_l^{(k)} = N_c - 1\}} I_{\{\epsilon_l^{(k)} > 0\}}] \\ + b_i^{(k)} d_l^{(1)} d_{l+1}^{(k)} &R(T_c + \epsilon_{l+1}^{(k)}) I_{\{p_{l+1}^{(k)} = 1\}} I_{\{\epsilon_{l+1}^{(k)} < 0\}}, \end{aligned} \tag{9}$$

for $l = iN_f, ..., (i+1)N_f - 2$. For $l = (i+1)N_f - 1$, $b_i^{(k)}$ in the third term is replaced by $b_{i+1}^{(k)}$.

Our aim is to obtain the probability distribution of $a^{(k)} = \sum_{l=iN_f}^{(i+1)N_f-1} a_l^{(k)}$. We will consider coded and uncoded systems separately at this point.

III. Coded Systems

Using the expressions in (5)-(9), the distribution of the MAI term from user k can be approximated as shown in the following lemma:

Lemma 3.1: *As $N \longrightarrow \infty$ and $\frac{N_f}{N_c} \longrightarrow c > 0$, $a^{(k)}$ in (5) is asymptotically normally distributed as*

$$a^{(k)} \sim \mathcal{N}(0\,,\, \gamma_2^{(k)} N_f / N_c), \tag{10}$$

where $\gamma_2^{(k)} = \mathrm{E}\{R^2(\epsilon^{(k)})\} + \mathrm{E}\{R^2(T_c - |\epsilon^{(k)}|)\}$.

Proof: See [4].

From (4) and (10), the BEP of the coded IR system conditioned on the timing jitter of user 1 can be approximated as follows:

$$P_{e|\boldsymbol{\epsilon}_i^{(1)}} \approx Q\left(\frac{\sqrt{\frac{E_1}{N_f}} \sum_{j=iN_f}^{(i+1)N_f-1} R(\epsilon_j^{(1)})}{\sqrt{\frac{1}{N_c}\sum_{k=2}^{N_u} E_k \gamma_2^{(k)} + N_f \sigma_n^2}} \right), \tag{11}$$

where $\boldsymbol{\epsilon}_i^{(1)} = [\epsilon_{iN_f}^{(1)} \ldots \epsilon_{(i+1)N_f-1}^{(1)}]$.

For large values of N_f, it follows from the Central Limit Theorem (CLT) that $\frac{1}{\sqrt{N_f}} \sum_{j=iN_f}^{(i+1)N_f-1} [R(\epsilon_j^{(1)}) - \mathrm{E}\{R(\epsilon_j^{(1)})\}]$ is approximately Gaussian. Then, using the relation $\mathrm{E}\{Q(X)\} = Q\left(\frac{\hat{\mu}}{\sqrt{1+\hat{\sigma}^2}}\right)$ for $X \sim \mathcal{N}(\hat{\mu}, \hat{\sigma}^2)$ [10], the unconditional BEP can be expressed approximately as follows:

$$P_e \approx Q\left(\frac{\sqrt{E_1}\mu}{\sqrt{\frac{E_1 \sigma^2}{N_f} + \frac{1}{N}\sum_{k=2}^{N_u} E_k \gamma_2^{(k)} + \sigma_n^2}} \right), \tag{12}$$

where $\mu = \mathrm{E}\{R(\epsilon_j^{(1)})\}$ and $\sigma^2 = \mathrm{Var}\{R(\epsilon_j^{(1)})\}$.

From the last expression, it is observed that the BEP decreases as N_f increases. In other words, the BEP is smaller for larger number of pulses per information symbol. We observe from (12) that the second term in the denominator, which is due to the MAI, depends on N_c and N_f only through their product $N = N_c N_f$. Therefore, the MAI has no effect on the trade-off between processing gains for a fixed total processing gain N. The only term that depends on the distribution of N between N_c and N_f is the first term in the denominator, which reflects the effect of timing jitter. This effect is mitigated by choosing small N_c, or large N_f, which means sending more pulses per information bit. Therefore, for a coded system, keeping N_f large helps to reduce BEP. Also note that in the absence of timing jitter, (12) reduces to

$$P_e = Q\left(\frac{\sqrt{E_1}}{\frac{1}{N}\sum_{k=2}^{N_u} E_k \gamma_2^{(k)} + \sigma_n^2} \right), \tag{13}$$

in which case there is no effect of processing gain parameters to BEP performance, as stated in [5].

IV. Uncoded Systems

For uncoded systems, the following lemma approximates the conditional probability distribution of $a^{(k)}$ in (5):

Lemma 4.1: *As $N \longrightarrow \infty$ and $\frac{N_f}{N_c} \longrightarrow c > 0$, $a^{(k)}$, given the information bit $b_i^{(k)}$, is approximately distributed as*

$$a^{(k)}|b_i^{(k)} \sim \mathcal{N}\left(\frac{N_f}{N_c} b_i^{(k)}\gamma_1^{(k)}\,,\, \frac{N_f}{N_c}\left[\gamma_2^{(k)} - \frac{(\gamma_1^{(k)})^2}{N_c} + \frac{\beta_1^{(k)}}{N_c^2} + \frac{\beta_2^{(k)}}{N_c^3}\right]\right), \tag{14}$$

where

$$\begin{aligned}\gamma_1^{(k)} &= \mathrm{E}\{R(\epsilon^{(k)})\} + \mathrm{E}\{R(T_c - |\epsilon^{(k)}|)\},\\ \gamma_2^{(k)} &= \mathrm{E}\{R^2(\epsilon^{(k)})\} + \mathrm{E}\{R^2(T_c - |\epsilon^{(k)}|)\},\\ \beta_1^{(k)} &= 2\mathrm{E}\{R(T_c - |\epsilon^{(k)}|)R(\epsilon^{(k)})\} - 2(\mathrm{E}\{R(T_c - |\epsilon^{(k)}|)\})^2\\ &\quad + 4\int_{-\infty}^{0} R(T_c + \epsilon^{(k)})p(\epsilon^{(k)})d\epsilon^{(k)}\\ &\quad \times \int_{0}^{\infty} R(T_c - \epsilon^{(k)})p(\epsilon^{(k)})d\epsilon^{(k)},\\ \beta_2^{(k)} &= 2(\mathrm{E}\{R(T_c - |\epsilon^{(k)}|)\})^2.\end{aligned} \tag{15}$$

Proof: See [4].

Note that for systems with large N_c, the distribution of $a^{(k)}$ given the information symbol $b_i^{(k)}$ can be approximately expressed as

$$a^{(k)}|b_i^{(k)} \sim \mathcal{N}\left(b_i^{(k)}\gamma_1^{(k)} N_f/N_c\,,\, \frac{N_f}{N_c}[\gamma_2^{(k)} - (\gamma_1^{(k)})^2/N_c]\right). \tag{16}$$

First consider a two-user system. For equiprobable information symbols ± 1, the BEP conditioned on the timing jitter of the first user can be shown to be

$$\mathrm{P}_{e|\boldsymbol{\epsilon}^{(1)}} \approx \frac{1}{2} Q\left(\frac{\frac{\sqrt{E_1}}{N_f}\sum_{j=iN_f}^{(i+1)N_f-1} R(\epsilon_j^{(1)}) + \frac{\sqrt{E_2}}{N_c}\gamma_1^{(2)}}{\sqrt{\frac{E_2}{N}[\gamma_2^{(2)} - (\gamma_1^{(2)})^2/N_c] + \sigma_n^2}}\right) + \frac{1}{2} Q\left(\frac{\frac{\sqrt{E_1}}{N_f}\sum_{j=iN_f}^{(i+1)N_f-1} R(\epsilon_j^{(1)}) - \frac{\sqrt{E_2}}{N_c}\gamma_1^{(2)}}{\sqrt{\frac{E_2}{N}[\gamma_2^{(2)} - (\gamma_1^{(2)})^2/N_c] + \sigma_n^2}}\right). \tag{17}$$

Then, for large N_f values, we can again invoke the CLT for $\frac{1}{\sqrt{N_f}}\sum_{j=iN_f}^{(i+1)N_f-1}[R(\epsilon_j^{(1)}) - \mu]$ and approximate the unconditional BEP as

$$P_e \approx \frac{1}{2}Q\left(\frac{\sqrt{E_1}\mu + \frac{\sqrt{E_2}}{N_c}\gamma_1^{(2)}}{\sqrt{\frac{E_1\sigma^2}{N}N_c + \frac{E_2}{N}[\gamma_2^{(2)} - (\gamma_1^{(2)})^2/N_c] + \sigma_n^2}}\right) + \frac{1}{2}Q\left(\frac{\sqrt{E_1}\mu - \frac{\sqrt{E_2}}{N_c}\gamma_1^{(2)}}{\sqrt{\frac{E_1\sigma^2}{N}N_c + \frac{E_2}{N}[\gamma_2^{(2)} - (\gamma_1^{(2)})^2/N_c] + \sigma_n^2}}\right). \tag{18}$$

For the multiuser case, assume that all interfering users have the same energy E and that their jitter sequences are i.i.d. from user to user. Then, the total MAI can be approximated by a zero mean Gaussian random variable for a sufficiently large number of users, N_u, and, after similar manipulation, the BEP

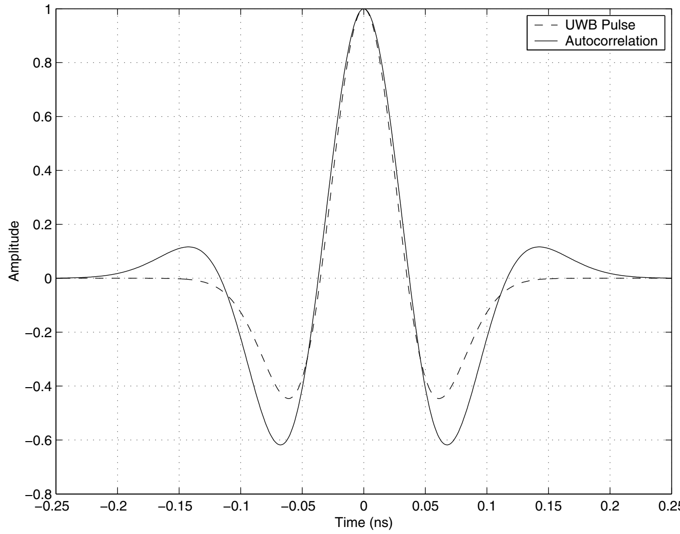

Fig. 2. UWB pulse and the autocorrelation function for $T_c = 0.25ns$.

can be expressed approximately as

$$P_e \approx Q\left(\frac{\sqrt{E_1}\mu}{\sqrt{\frac{E_1\sigma^2}{N}N_c + (N_u - 1)E\left(\frac{\gamma_2}{N} + \frac{\gamma_1^2}{N_c^2} - \frac{\gamma_1^2}{NN_c}\right) + \sigma_n^2}}\right), \tag{19}$$

where the user index k is dropped from $\gamma_1^{(k)}$ and $\gamma_2^{(k)}$ due to the identicality.

Considering (19), it is seen that for relatively small N_c values, the second term in the denominator, which is the term due to MAI, can become large and cause an increase in the BEP. Similarly, when N_c is large, the first term in the denominator can become significant and the BEP can become large again. Therefore, we expect to have an optimal N_c value. Intuitively, for small N_c values, the number of pulses per bit, N_f, is large. Therefore, we can have high BEP due to a large amount of MAI. As N_c becomes large, the MAI becomes more negligible. However, making N_c very large again causes an increase in BEP since N_f becomes small, in which case the effect of timing jitter becomes more significant. The optimal N_c (N_f) value can be approximated by using (19).

V. Simulation Results

In this section, the BEP performance of coded and uncoded systems is evaluated by conducting simulations for different values of the processing gains and the results are compared to the theoretical results. The UWB pulse[7] and the normalized autocorrelation function used in the simulations are as follows

[7] $w_{rx}(t) = w(t)/\sqrt{E_p}$ with $E_p = \int_{-\infty}^{\infty} w^2(t)dt$ is used as the received UWB pulse with unit energy.

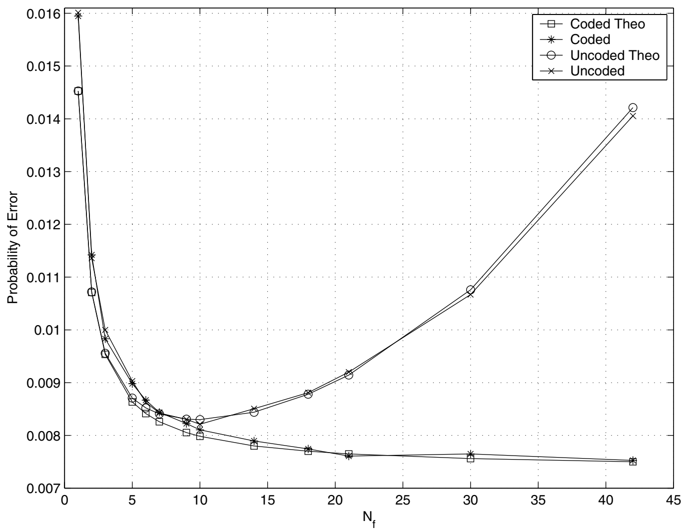

Fig. 3. BEP versus N_f for coded and uncoded systems with uniform jitter $\mathcal{U}[-25ps, 25ps]$, where $N = 630$, $N_u = 10$, $\sigma_n^2 = 0.1$ and $E_k = 1\ \forall k$.

[11]:

$$w(t) = \left(1 - \frac{4\pi t^2}{\tau^2}\right) e^{-2\pi t^2/\tau^2}, \quad (20)$$

$$R(\Delta t) = \left[1 - 4\pi(\frac{\Delta t}{\tau})^2 + \frac{4\pi^2}{3}(\frac{\Delta t}{\tau})^4\right] e^{-\pi(\frac{\Delta t}{\tau})^2}, \quad (21)$$

where $\tau = 0.125ns$ is used.

The timing jitter is modelled by the uniform distribution $\mathcal{U}[-25ps, 25ps]$ and T_c is chosen to be $0.25ns$. The total processing gain $N = N_c N_f$ is taken to be 630. Also all 10 users ($N_u = 10$) are assumed to be sending unit energy per symbol ($E_k = 1\ \forall k$) and $\sigma_n^2 = 0.1$.

Figure 3 shows the BEP of the coded and the uncoded system for different N_f values. It is seen that theoretical values match quite closely with the simulation results, especially when N_f gets larger, since the Gaussian approximation becomes better as N_f increases. For the coded system, the BEP decreases as N_f increases. Since the MAI has no effect on the BEP for a given value of N, only the effect of timing jitter needs to be considered. Because the effect of timing jitter is reduced for large N_f, the plots for coded system show a decrease in BEP as N_f increases. For the uncoded system, there is an optimal value of the processing gain that minimizes the BEP of the system. In this case, there are both the effect of timing jitter and the effect of MAI. The effect of timing jitter is mitigated by using large N_f while that of MAI is reduced by small N_f. The optimal value of the processing gains can be approximately calculated using (19).

VI. Conclusion

The trade-off between the two types of processing gain has been investigated in the presence of timing jitter. It is concluded that in an AWGN channel sending more pulses per bit decreases the BEP in a coded system since effect of MAI on the probability of error is fixed for a given value of total processing gain and the effect of timing jitter is reduced by sending more pulses. In an uncoded system, there is a trade-off between N_c and N_f, which reflects the effects of timing jitter and MAI. Optimal processing gains can be found by using an approximate closed form expression for the BEP. Current work focuses on the extension of the results to frequency selective channels [4].

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