On Geolocation Accuracy with Prior Information in Non-line-of-sight Environment

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***Abstract*— Non-line-of-sight (NLOS) geolocation becomes an important issue with the fast development of mobile communications in recent years. Several methods have been proposed to address this problem. However, a comprehensive study on the best geolocation accuracy that these methods may possibly achieve is called for. In [1], [2], we reported a unified analysis of the Cramer-Rao Lower Bound (CRLB) and achievable bounds applicable to NLOS geolocation, assuming no prior information on the mobile station (MS) position or NLOS induced paths is available. In practice, however, we often have some information about these parameters beforehand. In this paper, we derive a lower bound for the geolocation accuracy in the presence of such prior information, and explore its physical interpretation. Some numerical examples are discussed.**

I. Introduction

Geolocation in non-line-of-sight (NLOS) environment is an important topic in wireless communications. Several methods [3]–[7] have been proposed to mitigate NLOS effects in geolocation. However, there has been no systematic study reported regarding the best geolocation accuracy that these methods may possibly achieve, which should be of practical and theoretical interest. A complete analysis of NLOS geolocation with multipaths would be very complicated. To make the problem manageable, our current study focuses on a scenario in which a single (line-of-sight (LOS) or NLOS) propagation path exists for each base station (BS) and mobile station (MS) pair. In [1], we presented a unified treatment to obtain the Cramer-Rao Lower Bound (CRLB) for various NLOS geolocation approaches. Our further study showed, however, that the CRLB is not achievable in general. The achievable bound is then investigated in [2]. One interesting and common characteristic for the two bounds is that contribution of NLOS signals ought to be completely ignored, i.e., the bound for NLOS geolocation is equivalent to the one applicable to a situation in which only signals from LOS stations are processed. This results from the assumption that no prior statistics for the MS position or NLOS delays is available.

In practice, however, it is possible to acquire some statistical characteristics on delays of NLOS signals or an MS position beforehand. With such information, better positioning accuracy is reasonably expected. In this paper, we extend our previous study by incorporating the prior information to a lower bound for NLOS geolocation accuracy. To emphasize major points, we begin with a simple (albeit somewhat unrealistic) scenario that NLOS delays are independent Gaussian random variables. We then model the delays to be Gamma distributed, which is more realistic than Gaussian since a NLOS delay is always represented by a non-negative number. The situation with prior knowledge on the MS location is then addressed.

The rest of the paper is structured as follows. We present the problem formulation in Section II. In Section III, we consider the lower bound for geolocation accuracy with prior information for NLOS delays. Its physical significance is addressed. We then discuss, in Section IV, the issue with prior information for the MS position. Section V provides some numerical examples. We make a brief conclusion in the last section.

II. Problem Formulation

Let $\mathcal{B} = \{1, 2, \cdots, B\}$ be the set of indices of B base stations, whose locations are at $\left\{\underline{p_b} = (x_b, y_b),\ b \in \mathcal{B}\right\}$. Denote the set of the BS's that receive NLOS signals as $\mathcal{M} = \{k_1, k_2, \cdots, k_M\}$. We can assume $\mathcal{M} = \{1, 2, \cdots, M\}$ without loss of generality. The complement $\mathcal{L} = \mathcal{B} \setminus \mathcal{M}$ is the set of LOS stations, with its cardinality being $|\mathcal{L}| = B - M$. The parameter of our interest is the MS position $\underline{p} = (x, y)$, yet there are M additional unknown parameters, NLOS propagation induced path lengths, $\underline{l} = (l_1, l_2, \cdots, l_M)$. Thus, we define an $(M+2)$-dimensional vector $\underline{\theta} = (\underline{p},\ \underline{l})$. The *a priori* joint probability density of $\underline{\theta}$ is $p_{\underline{\theta}}(\underline{\theta})$. Let τ_b be the time delay of the signal at base station b (BS_b), specifically to be,

$$\tau_b = \frac{1}{c}\left\{\sqrt{(x_b - x)^2 + (y_b - y)^2} + l_b\right\}, \qquad (1)$$

where $l_b = 0$ if $b \in \mathcal{L}$, $c = 3 \times 10^8\ m/s$ is the speed of light.

The received signal at BS_b is

$$r_b(t) = A_b s(t - \tau_b) + n_b(t), \quad \text{for } b \in \mathcal{B}, \qquad (2)$$

where A_b is the signal amplitude for BS_b, $s(t)$ is the base-band waveform, and $n_b(t)$'s are independent complex-valued white Gaussian noise processes with spectral density $N_0/2$.

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The joint probability density function of observables $\{r_b(t),\ b \in \mathcal{B}\}$ conditioned on $\underline{\theta}$ is

$$f_{\underline{\theta}}(\underline{r}) \propto \prod_{b=1}^{B} \exp\left\{-\frac{1}{N_0}\int |r_b(t) - A_b s(t-\tau_b)|^2\, dt\right\}. \quad (3)$$

By casting the NLOS geolocation as a multi-parameter estimation problem, we wish to obtain a lower bound on the mean-square error in estimating the MS position $\underline{p}$. Denote the information matrix by $\mathbf{J}$, which is the Bayesian version [8] of the Fisher information matrix. The matrix $\mathbf{J}$ consists of two parts,

$$\mathbf{J} = \mathbf{J}_D + \mathbf{J}_P, \quad (4)$$

where the subscripts "D" and "P" denote the information due to the data $\underline{r}$ and the prior knowledge $p_{\underline{\theta}}(\underline{\theta})$, respectively.

$$\mathbf{J}_D = E\left[\frac{\partial}{\partial\underline{\theta}}\log f_{\underline{\theta}}(\underline{r}) \cdot \left(\frac{\partial}{\partial\underline{\theta}}\log f_{\underline{\theta}}(\underline{r})\right)^T\right], \quad (5)$$

where $\frac{\partial}{\partial\underline{\theta}}\log f_{\underline{\theta}}$ is an $(M+2)$ column vector, symbol "T" designates transpose, and the expectation is taken over $\underline{r}$ and $\underline{\theta}$.

$$(\mathbf{J}_P)_{ij} \equiv -E\left(\frac{\partial^2}{\partial\theta_i\partial\theta_j}\ln p_{\underline{\theta}}(\underline{\theta})\right), \quad \text{for } 1 \le i,j \le (M+2), \quad (6)$$

where the expectation is over $\underline{\theta}$.

The relation between the covariance matrix of $\underline{\theta}$, Cov($\underline{\theta}$), and $\mathbf{J}$ is

$$\text{Cov}(\underline{\theta}) \ge \mathbf{J}^{-1}, \quad (7)$$

where the inequality means that the matrix $(\text{Cov}(\underline{\theta}) - \mathbf{J}^{-1})$ is non-negative definite. Note that this provides a lower bound on the mean-square errors, specifically to be

$$E(\hat{\theta}_i - \theta_i)^2 \ge (\mathbf{J}^{-1})_{ii}, \quad \text{for } 1 \le i \le (M+2). \quad (8)$$

The quantities with $i = 1, 2$ are for the MS position. Compared with the CRLB which is conditioned on specific values of the parameters to be estimated, the Bayesian bound ($\mathbf{J}^{-1}$) utilizes the *a priori* probability density of the parameters and provides a "global bound" that does not depend on the values on a specific trial.

III. Lower bound with prior information on NLOS propagation

We first look into a simple scenario that the NLOS delays, $\underline{l}$, are known to be independently Gaussian distributed. As we will see soon, it leads naturally to the major point of how the prior information may enhance geolocation precision. The derivation is then generalized to some realistic distribution, e.g. Gamma distribution, since the NLOS delays are non-negative in practice. A lower bound with the statistics of the MS position is complicate and left to the next section.

A. NLOS delays of Gaussian distribution

We divide the parameter vector $\underline{\theta}$ into a nonrandom and random components: the MS position $\underline{p}$ and the NLOS delays $\underline{l}$, respectively. The parameters in $\underline{l}$ have independent Gaussian distribution with mean $\underline{u}_l$ and a covariance matrix $\mathbf{\Sigma}_l$

$$\mathbf{\Sigma}_l = \begin{pmatrix} \sigma_{l_1}^2 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \sigma_{l_M}^2 \end{pmatrix}. \quad (9)$$

The expectation in Eqs. (5) and (6) for $\mathbf{J}_D$ and $\mathbf{J}_P$ now is taken over $\underline{r}$ and $\underline{l}$, and $\underline{l}$, respectively [8]. It is straightforward to obtain

$$\mathbf{J}_P = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_l^{-1} \end{pmatrix}. \quad (10)$$

To evaluate $\mathbf{J}_D$, we rewrite Eq. (5) as

$$\mathbf{J}_D = E\left(\mathbf{J}_{\underline{\theta}}\right), \quad (11)$$

where the expectation is on $\underline{l}$ only. The matrix $\mathbf{J}_{\underline{\theta}}$ is the Fisher information matrix conditioned on $\underline{\theta}$, i.e.,

$$\mathbf{J}_{\underline{\theta}} = E_{\underline{\theta}}\left[\frac{\partial}{\partial\underline{\theta}}\log f_{\underline{\theta}}(\underline{r}) \cdot \left(\frac{\partial}{\partial\underline{\theta}}\log f_{\underline{\theta}}(\underline{r})\right)^T\right], \quad (12)$$

where the expectation is over observables $\underline{r}$ conditioned on $\underline{\theta}$.

We have shown in [1] that

$$\mathbf{J}_{\underline{\theta}} = \mathbf{H} \cdot \mathbf{J}_{\underline{\tau}} \cdot \mathbf{H}^T, \quad (13)$$

where

$$\mathbf{H} = \begin{pmatrix} \frac{\partial\tau_1}{\partial x} & \cdots & \frac{\partial\tau_M}{\partial x} & \cdots & \frac{\partial\tau_B}{\partial x} \\ \frac{\partial\tau_1}{\partial y} & \cdots & \frac{\partial\tau_M}{\partial y} & \cdots & \frac{\partial\tau_B}{\partial y} \\ \frac{\partial\tau_1}{\partial l_1} & \cdots & \frac{\partial\tau_M}{\partial l_1} & \cdots & \frac{\partial\tau_B}{\partial l_1} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial\tau_1}{\partial l_M} & \cdots & \frac{\partial\tau_M}{\partial l_M} & \cdots & \frac{\partial\tau_B}{\partial l_M} \end{pmatrix}, \quad (14)$$

an $(M+2) \times B$ matrix, and $\mathbf{J}_{\underline{\tau}}$ is the Fisher information matrix conditioned on $\underline{\tau}$,

$$\mathbf{J}_{\underline{\tau}} = E_{\underline{\tau}}\left[\frac{\partial}{\partial\underline{\tau}}\log f_{\underline{\tau}}(\underline{r}) \cdot \left(\frac{\partial}{\partial\underline{\tau}}\log f_{\underline{\tau}}(\underline{r})\right)^T\right]. \quad (15)$$

The matrix $\mathbf{H}$ contains the geometric relation among the MS and BS's. It can be decomposed into a NLOS and LOS components

$$\mathbf{H} = \frac{1}{c} \cdot \begin{pmatrix} \mathbf{H}_{NL} & \mathbf{H}_L \\ \mathbf{I}_M & \mathbf{0} \end{pmatrix}, \quad (16)$$

where $\mathbf{I}_M$ is an identity matrix of order M, $\mathbf{H}_{NL}$ and $\mathbf{H}_L$ are $2 \times M$ and $2 \times (B-M)$ matrices, respectively, given by

$$\mathbf{H}_{NL} = \begin{pmatrix} \cos\phi_1 & \cdots & \cos\phi_M \\ \sin\phi_1 & \cdots & \sin\phi_M \end{pmatrix}, \quad \text{and}$$

$$\mathbf{H}_L = \begin{pmatrix} \cos\phi_{M+1} & \cdots & \cos\phi_B \\ \sin\phi_{M+1} & \cdots & \sin\phi_B \end{pmatrix},$$

and angle ϕ_b is determined by

$$\phi_b = \tan^{-1} \frac{y - y_b}{x - x_b}.$$

The subscripts "NL" and "L" denote the quantities for NLOS and LOS stations, respectively. Similarly,

$$\mathbf{J}_{\underline{\tau}} = \begin{pmatrix} \mathbf{\Lambda}_{NL} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_L \end{pmatrix}, \tag{17}$$

where $\mathbf{\Lambda}_{NL}$ and $\mathbf{\Lambda}_L$ are diagonal matrices of order M and $(B - M)$, respectively, as

$$\mathbf{\Lambda}_{NL} = \begin{pmatrix} \lambda_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \lambda_M \end{pmatrix}, \text{ and}$$

$$\mathbf{\Lambda}_L = \begin{pmatrix} \lambda_{M+1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \lambda_B \end{pmatrix}.$$

Their entries are

$$\lambda_b = 8\pi^2\beta^2 \cdot R_b, \quad \text{for } b \in \mathcal{B}, \tag{18}$$

where R_b is the signal-to-noise ratio (SNR) of the received signal at BS_b, i.e.,

$$R_b = \frac{\int |A_b s(t)|^2 dt}{N_0},$$

and the effective bandwidth of the signal waveform, β, is determined by

$$\beta^2 = \int f^2 |S(f)|^2 df.$$

$S(f)$ is the Fourier transform of $s(t)$.

Note that matrices $\mathbf{H}$ and $\mathbf{J}_{\underline{\tau}}$ in Eqs. (16) and (17) are independent of $\underline{l}$. We can remove the expectation sign in Eq. (11), i.e.,

$$\mathbf{J}_D = \mathbf{J}_{\underline{\theta}}. \tag{19}$$

With Eqs. (13), (16) and (17), Eq. (19) becomes

$$\mathbf{J}_D = \frac{1}{c^2} \cdot \begin{pmatrix} \mathbf{H}_{NL}\mathbf{\Lambda}_{NL}\mathbf{H}_{NL}^T + \mathbf{H}_L\mathbf{\Lambda}_L\mathbf{H}_L^T & \mathbf{H}_{NL}\mathbf{\Lambda}_{NL} \\ \mathbf{\Lambda}_{NL}\mathbf{H}_{NL}^T & \mathbf{\Lambda}_{NL} \end{pmatrix}. \tag{20}$$

Up to now we have evaluated both ingredients for the matrix $\mathbf{J}$ in Eq. (4). Denote the Bayesian bound as

$$\mathbf{L} \equiv \mathbf{J}^{-1} = (\mathbf{J}_D + \mathbf{J}_P)^{-1}. \tag{21}$$

Since the MS position accuracy is of our major concern, we only consider $\mathbf{L}_{2\times2}$, which is the first 2×2 diagonal matrix of $\mathbf{L}$. The explicit expression of $\mathbf{L}_{2\times2}$ is complex (see Appendix). We choose to obtain its lower and upper bounds instead, for a clear physical interpretation. We derive in Appendix that

$$c^2 \left(\mathbf{H}_{NL}\mathbf{\Lambda}_{NL}\mathbf{H}_{NL}^T + \mathbf{H}_L\mathbf{\Lambda}_L\mathbf{H}_L^T\right)^{-1} \le \mathbf{L}_{2\times2} \le c^2 \left(\mathbf{H}_L\mathbf{\Lambda}_L\mathbf{H}_L^T\right)^{-1}. \tag{22}$$

The lower bound is attained when $\sigma_{l_m}^2 \longrightarrow 0$, for $1 \le m \le M$, i.e., we know the exact NLOS path length, in which NLOS stations are treated as LOS ones. On the other hand, the upper bound is achieved when all $\sigma_{l_m}^2 \longrightarrow +\infty$, which means we have no such prior information. We may notice the upper bound relies only on the LOS signals, which is reduced to the CRLB [2].

B. NLOS delays of Gamma distribution

As we know that the NLOS induced path length is always positive, it is more reasonable to model $\underline{l}$ to be independently Gamma distributed, i.e.,

$$G(\alpha_m, q_m) = \frac{\alpha_m^{q_m}}{\Gamma(q_m)} \exp(-\alpha_m \cdot l_m) \cdot l_m^{q_m - 1}, \quad \text{for } m \in \mathcal{M}, \tag{23}$$

where $q_m > 2$ and $\alpha_m > 0$. The balance between q_m and α_m controls the decay and spread pattern of the probability density function.

Substitute Eq. (23) into Eq. (6), we obtain

$$\mathbf{J}_P = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Pi}_l^{-1} \end{pmatrix}, \tag{24}$$

where

$$\mathbf{\Pi}_l^{-1} = \begin{pmatrix} \frac{\alpha_1^2}{q_1 - 2} & & & \mathbf{0} \\ & \frac{\alpha_2^2}{q_2 - 2} & & \\ & & \ddots & \\ \mathbf{0} & & & \frac{\alpha_M^2}{q_M - 2} \end{pmatrix}. \tag{25}$$

Compare Eqs. (25) and (10), we realize that, for fixed α_m, $q_m \longrightarrow 2$ is equivalent to $\sigma_{l_m} \longrightarrow 0$, and $q_m \longrightarrow +\infty$ is equivalent to $\sigma_{l_m} \longrightarrow +\infty$ in Eq. (9). Therefore, an almost identical conclusion as Gaussian case applies for Gamma distribution. Specifically, Eq. (22) still holds. For fixed α_m, the lower bound is attained when $q_m \longrightarrow 2$, for $1 \le m \le M$. The upper bound is achieved when all $q_m \longrightarrow +\infty$.

It is clear that a similar derivation can be extended to any other distributions for NLOS delays.

IV. Lower bounds with prior information on the MS position

Suppose that we also have some prior statistics on the MS position. For an example, $\underline{p}$ are independent Gaussian distributed with mean $\underline{u}_p$ and a covariance matrix

$$\mathbf{\Sigma}_p = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}. \tag{26}$$

It is difficult to acquire an analytical result of the expectation over $\underline{p}$ in Eq. (6). Thus we only make a rough approximation: if the diagonal terms of the lower bound in Eq. (22) is larger than that of $\mathbf{\Sigma}_p$, we set $\mathbf{L}_{2\times2} = \mathbf{\Sigma}_p$; if the diagonal terms of the upper bound is smaller than that of $\mathbf{\Sigma}_p$, the extra MS information is ignored.

V. Numerical examples

We provide some numerical examples in this section. We simulate a cellular CDMA system, as shown in Figure 1, with the cell radius of 1000m. The bandwidth of CDMA signals is $W = 5$Mcps. The relation between W and the effective bandwidth β is derived as

$$\beta = \frac{W}{\sqrt{3}}. \tag{27}$$

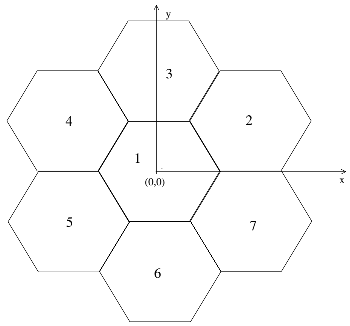

Fig. 1. A Cellular system with seven base stations. The cell radius is 1000m.

The SNR is set to be 3dB when the distance between the MS and BS is 2000m.

The Figure 2 shows the Bayesian bounds for the NLOS geolocation varies with NLOS induced delays, together with its lower and upper bound. No prior information on the MS position is assumed here. The BS_1 and BS_2 are NLOS BS's, while the other five stations receive LOS signals. The propagation loss factor is 2 for LOS (free space) and 4 for NLOS paths. As we may notice from the figure, the lower and upper bound becomes close when NLOS delays is larger. The reason is that the NLOS signals become weaker and contain "less information". The other trend is that when the variance of the NLOS delays is smaller, i.e., we have more accurate knowledge on $\underline{l}$, the Bayesian bound converges to the lower bound.

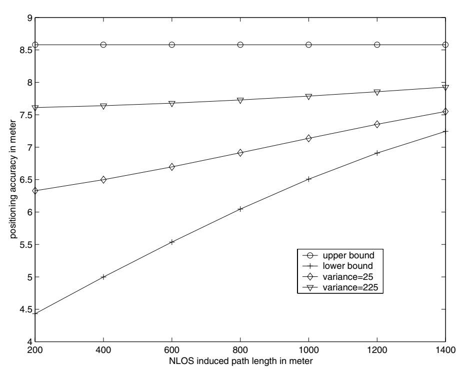

Fig. 2. MS positioning accuracy vs. NLOS induced path length. MS is located at (500m,700m). Propagation loss factor is 2 for LOS (free space) and 4 for NLOS path. BS1 and BS2 are NLOS BS's, while others are LOS stations.

VI. Conclusions

In this paper, we presented a lower bound for NLOS geolocation accuracy with prior information on NLOS delays and the MS position. Its physical interpretation has been explored. The lower bound provides an effective criterion to evaluate performance of various geolocation algorithms.

Appendix: Derivation of Eq. (22)

Rewrite $\mathbf{L}$ in Eq. (21) explicitly as

$$\mathbf{L} = \begin{pmatrix} \frac{1}{c^2}\mathbf{H}_{NL}\mathbf{\Lambda}_{NL}\mathbf{H}_{NL}^T + \frac{1}{c^2}\mathbf{H}_L\mathbf{\Lambda}_L\mathbf{H}_L^T & \frac{1}{c^2}\mathbf{H}_{NL}\mathbf{\Lambda}_{NL} \\ \frac{1}{c^2}\mathbf{\Lambda}_{NL}\mathbf{H}_{NL}^T & \frac{1}{c^2}\mathbf{\Lambda}_{NL} + \mathbf{\Sigma}_l^{-1} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{C} \end{pmatrix}^{-1}, \quad (28)$$

where matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are defined as in the above equation. It takes some calculation to obtain

$$\mathbf{L} = \begin{pmatrix} \mathbf{A}^{-1} + \mathbf{F}\mathbf{W}^{-1}\mathbf{F}^T & -\mathbf{F}\mathbf{W}^{-1} \\ -\mathbf{W}^{-1}\mathbf{F}^T & \mathbf{W}^{-1} \end{pmatrix}, \quad (29)$$

where

$$\mathbf{W} = \mathbf{C} - \mathbf{B}^T\mathbf{A}^{-1}\mathbf{B}, \ \ \mathbf{F} = \mathbf{A}^{-1}\mathbf{B}, \quad (30)$$

and the inverses that occur in the expressions exist [9]. Thus,

$$\begin{aligned} \mathbf{L}_{2\times 2} &= \mathbf{A}^{-1} + \mathbf{F}\mathbf{W}^{-1}\mathbf{F}^T \\ &= c^2\left(\mathbf{H}_{NL}\mathbf{\Lambda}_{NL}\mathbf{H}_{NL}^T + \mathbf{H}_L\mathbf{\Lambda}_L\mathbf{H}_L^T\right)^{-1} \\ &\quad + \mathbf{F}^T\left(\frac{1}{c^2}\mathbf{\Lambda}_{NL} + \mathbf{\Sigma}_l^{-1} - \mathbf{B}^T\mathbf{A}^{-1}\mathbf{B}\right)^{-1}\mathbf{F}, \end{aligned} \quad (31)$$

We have shown in [2] that the matrix $\frac{1}{c^2}\mathbf{\Lambda}_{NL} - \mathbf{B}^T\mathbf{A}^{-1}\mathbf{B}$ is positive definite. With $\mathbf{\Sigma}_l^{-1} \geq 0$, it is not difficult to verify

$$\begin{aligned} & c^2\left(\mathbf{H}_{NL}\mathbf{\Lambda}_{NL}\mathbf{H}_{NL}^T + \mathbf{H}_L\mathbf{\Lambda}_L\mathbf{H}_L^T\right)^{-1} \leq \mathbf{L}_{2\times 2} \\ & \leq c^2\left(\mathbf{H}_{NL}\mathbf{\Lambda}_{NL}\mathbf{H}_{NL}^T + \mathbf{H}_L\mathbf{\Lambda}_L\mathbf{H}_L^T\right)^{-1} \\ & + \mathbf{F}^T\left(\frac{1}{c^2}\mathbf{\Lambda}_{NL} - \mathbf{B}^T\mathbf{A}^{-1}\mathbf{B}\right)^{-1}\mathbf{F} \\ & = c^2\left(\mathbf{H}_L\mathbf{\Lambda}_L\mathbf{H}_L^T\right)^{-1}, \end{aligned} \quad (32)$$

where the last equality was proved in [2].

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